Volume Averaging Technique in numerical modelling of floating deep mixed columns in soft soils

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ABSTRACT: The deformation behaviour of deep mixed columns in anisotropic soft soil is a three-dimensional problem which has to be considered adequately in numerical modelling. However, in literature simplifications are often to be found due to computational costs of fully coupled 3D analyses by either modifying geometry of the problem or material parameters in 2D-plainstrain or -axisymmetric conditions. This paper uses an enhanced 2D volume averaging technique for numerical modelling of deep mixed columns. The method enables mapping the 3D problem in two dimensions, and yet modelling the two constituents (column and soft soil) appropriately within a homogenized material. The performance of the technique is demonstrated by considering unit cell simulations of floating columns in soft soil, in which the results from 2D finite element simulations utilizing volume averaging technique are compared against conventional and fully 3D coupled finite element analyses. It is shown that the quality of the match is very good but depending on the number of columns and stiffness ratio between columns and soft soil.

1 INTRODUCTION

The properties of very soft clays, silts and organic soils can be improved with deep mixing, a soil improvement technique in which stabilizing agents, such as lime and/or cement are mixed into the soil in situ by using auger-type mixing tools. Deep mixed columns are nowadays extensively used to reduce settlements and to improve the overall stability of road and railway embankments and foundations on soft soils.

Whilst the most typical applications are embankments, increasingly the method is used under foundations. The first category of applications is very common in Scandinavian countries, whilst the latter application has been pioneered in Poland as a cost effective alternative for piling. Due to economic, sustainability and environmental reasons there is an increasing interest to the technique in the rest of Europe. With deep mixing, the strength and deformation properties of the soft soil can be improved and the risk of large horizontal or vertical deformations is substantially reduced.

Numerical methods, such as finite element (FE) analysis, can be used as an alternative to conventional design methods. They are particularly suitable for serviceability limit state design. FE analyses allow adopting advanced constitutive models that take the complex stress-strain behaviour of natural soil and stabilized columns into account. The problems involving a grid of circular columns under an embankment or a strip footing are fully three-dimensional problems. As 3D analyses are computationally very expensive an enhanced 2D technique using the so-called volume averaging technique (VAT) is adopted. The basic idea is to describe the column-improved ground as a homogenized composite material and map the true 3D problem into 2D. Once the constitutive relations of both composites are defined, the response of the column improved ground can be studied in two dimensions subject to arbitrary loading and boundary conditions.

In this paper the volume averaging technique by Vogler & Karstunen (2007, 2009) is used in which two advanced constitutive models for soft soil and deep mixed column were implemented: the S-CLAY1S model (Karstunen et al. 2005) to represent the soft soil and the MNhard model (Benz 2007) to represent the deep mixed columns.

The performance of the volume averaging technique is demonstrated by comparing the results of discrete 2D axisymmetric and full 3D models of unit cell simulations of floating deep mixed columns, i.e. column and soil are modelled separately, with enhanced 2D analyses using the volume averaging technique.

2 VOLUME AVERAGING TECHNIQUE

2.1 *Introduction and fundamental assumptions*

The basic idea of the volume averaging technique is to model the periodic system as a homogenous material instead of modelling columns and natural soil separately (Fig. 1). The principles adopted by Vogler & Karstunen (2007, 2009) are based on the ideas of Schweiger & Pande (1986), further refined by Lee & Pande (1998). The formulation has been extended to three dimensions and a new solution routine has been developed to cope with the highly non-linear constitutive models.

Within the volume averaging technique a periodic distribution of the columns in the natural soil is assumed. Furthermore perfect bonding, in other words no slip between natural soil and columns, is assumed. The method allows for adopting any elasto-plastic constitutive model to the two constituents: natural and improved soil. Local equilibrium between soil and column as well as compatibility and validity of the constitutive relations are satisfied through stress/strain redistribution within a sub-iterating procedure.

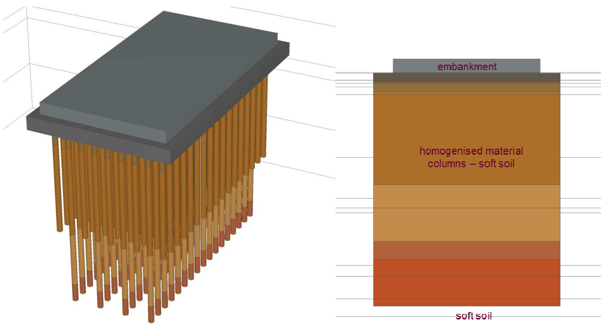

Figure 1. Discrete (left) and homogenised (right) representation of embankment problem.

2.2 *Equivalent material stiffness matrix*

Homogenisation is carried out by determining the strain increment and the stress increment in the homogenised equivalent material according to the following averaging rules:

$$\dot{\boldsymbol{\sigma}}^{eq} = \Omega_s \dot{\boldsymbol{\sigma}}^{s} + \Omega_c \dot{\boldsymbol{\sigma}}^{c} \tag{1a}$$

$$\dot{\boldsymbol{\varepsilon}}^{eq} = \Omega_s \dot{\boldsymbol{\varepsilon}}^{s} + \Omega_c \dot{\boldsymbol{\varepsilon}}^{c} \tag{1b}$$

where Ω is a volume fraction. The superscripts eq, s and c refer to the homogenized material, the soil and the column material, respectively. $\dot{\sigma}$ and $\dot{\varepsilon}$ are the (total) stress and strain rate tensors, respectively.

In the following it is assumed that the y-axis is in the vertical direction. The initial assumption of local equilibrium between the soil and the column material in each integration point can be formulated with the following equilibrium conditions, which assure that there is no stress discontinuity between soil and column material in terms of radial and shear stress, see also Figure 2:

$$\dot{\sigma}_x^{eq} = \dot{\sigma}_x^{s} = \dot{\sigma}_x^{c} \tag{2a}$$

$$\dot{\sigma}_z^{eq} = \dot{\sigma}_z^{s} = \dot{\sigma}_z^{c} \tag{2b}$$

$$\dot{\tau}_{xy}^{eq} = \dot{\tau}_{xy}^{s} = \dot{\tau}_{xy}^{c} \tag{2c}$$

$$\dot{\tau}_{yz}^{eq} = \dot{\tau}_{yz}^{s} = \dot{\tau}_{yz}^{c} \tag{2d}$$

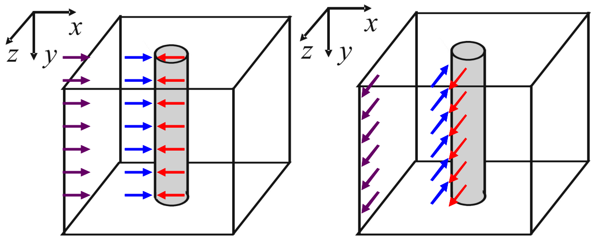

Figure 2. Local equilibrium conditions between column and soil for radial (left) and shear stress (right).

Furthermore, perfect bonding between the columns and the soft soil is assumed, and hence no slip is permitted between the two materials. This can be achieved with the following kinematic conditions, see also Figure 3:

$$\dot{\varepsilon}_y^{eq} = \dot{\varepsilon}_y^{s} = \dot{\varepsilon}_y^{c} \tag{3a}$$

$$\dot{\gamma}_{zx}^{eq} = \dot{\gamma}_{zx}^{s} = \dot{\gamma}_{zx}^{c} \tag{3b}$$

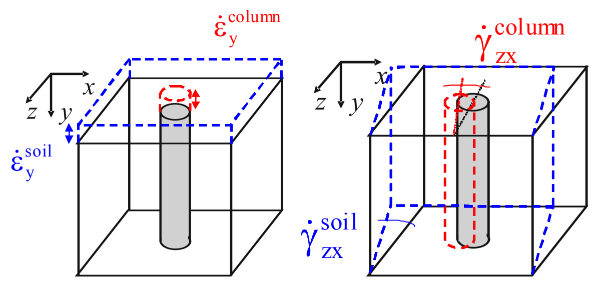

Figure 3. Kinematic conditions of perfect bonding between column and soil for axial (left) and shear strain (right).

The constitutive equations for the constituents can be described in terms of effective stress increments as:

$$\left(\dot{\boldsymbol{\sigma}}^{s}\right)' = \mathbf{D}^{s}\,\dot{\boldsymbol{\varepsilon}}^{s} \tag{4a}$$

$$\left(\dot{\boldsymbol{\sigma}}^{c}\right)' = \mathbf{D}^{c}\,\dot{\boldsymbol{\varepsilon}}^{c} \tag{4b}$$

where $\mathbf{D}^{c,s}$ represent the appropriate elasto-plastic material (or elastic) stiffness matrices for the soil and the columns, expressed naturally in terms of effective stress.

In principle any elasto-plastic constitutive law can be chosen for either of the two constituents. Here, the soft soil has been modelled with the S-CLAY1S model (Karstunen et al., 2005) and the columns with the MNhard model (Benz, 2007), which are described in Section 3. Considering the averaging rules (Eq. 1), the equilibrium and kinematic conditions (Eq. 2 & 3) and the constitutive relations (Eq. 4), the constitutive relation for the averaged material can be written as:

$$\left(\dot{\boldsymbol{\sigma}}^{eq}\right)' = \mathbf{D}^{eq}\dot{\boldsymbol{\varepsilon}}^{eq} \tag{5}$$

with the equivalent stiffness matrix defined as

$$\mathbf{D}^{eq} = \Omega_s \mathbf{D}^s \mathbf{S}_1^s + \Omega_c \mathbf{D}^c \mathbf{S}_1^c \tag{6}$$

3 CONSTITUTIVE MODELING

3.1 *Modelling of soft soil with S-CLAY1S*

The S-CLAY1 model (Wheeler et al. 2003) is a critical state model that is based on the Modified Cam Clay (MCC) model. Similarly to the MCC model, S-CLAY1 is assuming isotropic elasticity within the yield surface. Additionally to the MCC model, S-CLAY1 is capable of simulating anisotropic soil behaviour, induced by the geological and mineralogical history, and the subsequent loading of natural soil deposits. This anisotropy is modelled by inclining the yield surface to represent experimentally observed yield points for soft clays. Furthermore, changes of anisotropy due to plastic straining are taken into account by an additional hardening law. S-CLAY1 is capable of simulating accurate yield points and the correct development of volumetric strains and shear strains for reconstituted clays, as demonstrated by Karstunen & Koskinen (2008).

The S-CLAY1S model (Karstunen et al. 2005) is an extension to the S-CLAY1 model which allows additionally for modelling destructuration of bonds as necessary for natural soft clays. The effect of bonding is described by an "intrinsic yield surface", which is of the same shape and inclination as the yield surface of the natural soil (Fig. 4), but with a size p'_{mi} that is related to p'_m of the natural clay by the amount of bonding x ($p'_m = p'_{mi}(1 + x)$). The initial value for the amount of bonding x_0 can be estimated based on the sensitivity of the clay.

The values for the model parameters for the soft soils considered in the unit cell simulation, the Swedish Noedinge clay are given in Tables 1 to 3. For definitions of the soil constant and state variables, the reader should refer to Karstunen et al. (2005).

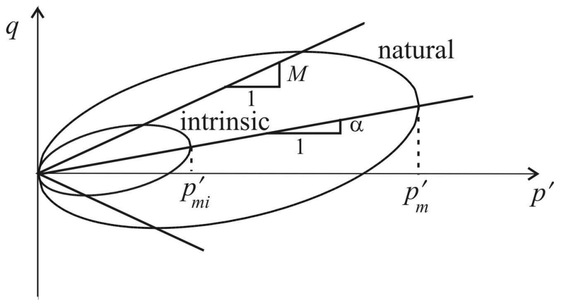

Figure 4. Yield surface of S-CLAY1S (triaxial stress space).

3.2 *Modelling of deep mixed columns*

The deep mixed columns have been modelled with the MNhard model (Benz 2007). MNhard is formulated in the classical theory of plasticity. Different stress-dependent stiffnesses are assumed for both elastic unloading/reloading and primary shear loading. The hyperbolic stress-strain relationship for primary loading is defined by the secant stiffness modulus E_{50} (Fig. 5). E_{50} (and similarly E_{ur} representing unloading/reloading) are defined stress dependent:

$$E_{50} = E_{50}^{ref}\left(\frac{c'\cos\varphi' + \sigma_3'\sin\varphi'}{c'\cos\varphi' + p'_{ref}\sin\varphi'}\right)^m \tag{7}$$

where E_{50}^{ref} is the secant stiffness modulus at isotropic reference pressure p'_{ref}, c' and ϕ' are cohesion and friction angle, σ'_3 is the effective minor principle stress and the power m is defining the amount of stress dependency. Failure criterion of the MNhard model is the Matsuoka-Nakai failure criterion, which is closer to the observed failure behaviour of granular materials than Mohr-Coulomb. However, the pre-failure behaviour for serviceability limit states is not affected by the failure criterion.

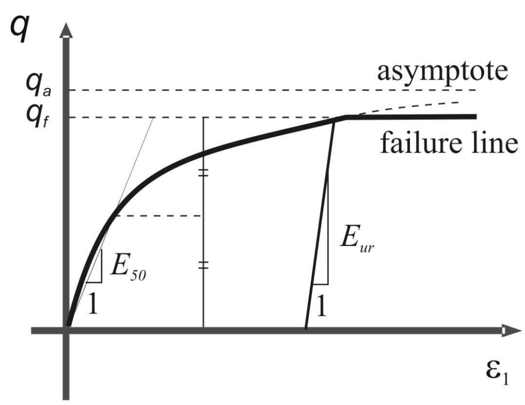

Figure 5. Hyperbolic stress-strain relationship in primary loading for a standard drained triaxial test.

As there are not many triaxial tests done on in-situ column material, the parameter values for deep mixed columns have been chosen to correspond to a series of triaxial tests (Aalto, 2003) performed on excavated columns, which have been fabricated in soft Finnish clay. The input parameters for the column are given in Table 4 using the equation $E = 2G(1 + \nu')$ based on the elasticity theory where G is the shear modulus.

Table 1. Initial values for state parameters and K_0.

Layer	e_0	OCR	K_0	α_0	x_0
Soft clay	2.9	1.25	0.56	0.458	10

Table 2. Conventional soil constants.

Layer	γ [kN/m³]	κ	ν'	λ	M
Soft clay	15	0.02	0.15	1.15	1.2

Table 3. Additional soil constants.

Layer	λ_i	β	μ	a	b
Soft clay	0.275	0759	30	10	0.2

Table 4. Input parameters for the columns.

Layer	γ [kN/m³]	G_{50}^{ref} [kPa]	G_{ur}^{ref} [kPa]	m	φ'	c' [kPa]	ν'
Column	15	4444	10000	0.8	35	15	0.35

Reference stress for stiffness, $p'_{ref} = 100$ kPa.

Table 5. Input parameters for the Dry Crust.

Layer	γ [kN/m³]	E [kPa]	φ'	c' [kPa]	ν'
Dry crust	15	750	30	6	0.15

3.3 *Solution strategy and implementation into finite element code*

The volume averaging technique is implemented into the PLAXIS (Brinkgreve et al. 2010) finite element code using an implicit backward Euler integration scheme. Because both constituents exhibit highly non-linear behaviour and an implicit integration scheme is used, a sub-iteration scheme was necessary: If the initially predicted internal strain distribution between soft clay and improved columns leads to an violation of the equilibrium conditions (Eq. 2), the strains between this two materials are redistributed in an iterative scheme until equilibrium is satisfied (more details in Vogler & Karstunen, 2007).

4 UNIT CELL SIMULATION

4.1 *2D and 3D unit cell FE Modell and analysis*

The performance of the volume averaging technique is demonstrated with the unit cell simulation of floating deep mixed columns. The soft soil is improved with lime-cement column of 10 m length and 0.6 m diameter and a centre to centre spacing of 1.0 m and 1.5 m, resulting in an improvement ratio, i.e. volume ratio, of Ω_c = 28.3% and 12.6% respectively.

Furthermore, the performance of the volume averaging technique is demonstrated by varying the stiffness of the deep mixed column with values of $G_{ur,ref}$ = 10, 20 and 50 MPa with a constant ratio $G_{50,ref} = G_{ur,ref}$ / 2.25. The influence of an embankment was modelled using a distributed load of 10 kPa and 50 kPa above a layer of dry crust with 1 m height. Full three-dimensional and two-dimensional discrete axisymmetric finite element calculations have been compared with enhanced axisymmetric calculations using volume averaging. The 3D mesh contains about 4,500 15-noded wedge-elements, whereas for the axisymmetric analysis 480 15-noded triangles were sufficient, see also Figure 6. For both calculations first the stresses were initialized using a K_0-procedure. Then the embankment load was brought up undrained, before simulating consolidation to a maximum remaining excess pore water pressure of 1 kPa.

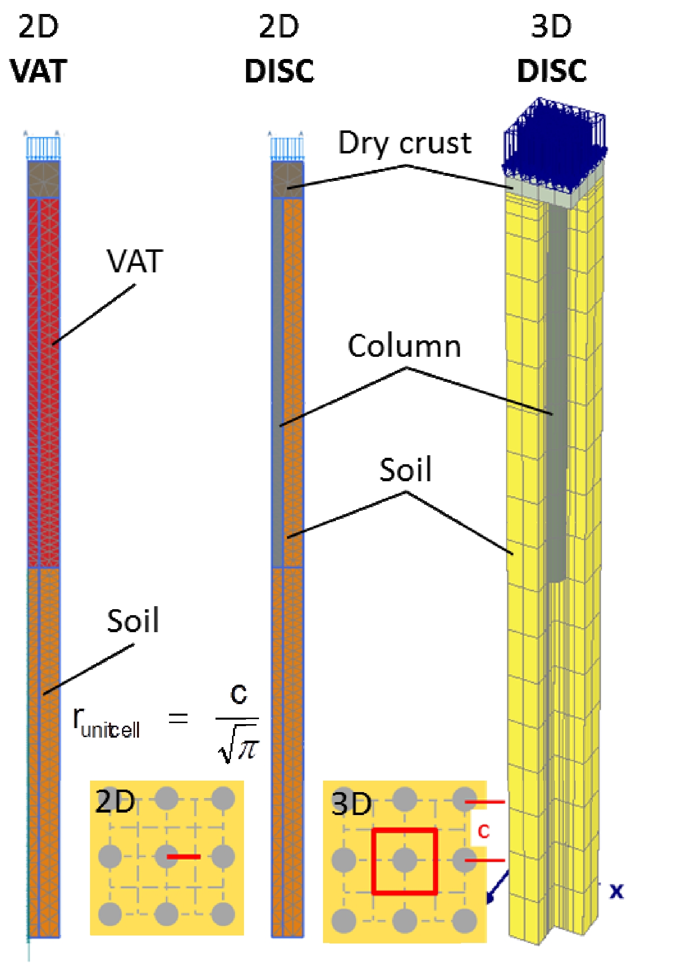

Figure 6. Unit cell FE models (2D and 3D) of floating columns.

4.2 Local equilibrium conditions

Local equilibrium conditions (Eq. 2) between column and soft soil are fulfilled if radial stresses of column and soil are equal at a virtual column-soil interface.

The radial stresses of column (C) and soil (S) from discrete modeling of the unit cell with axisymmetric (2D-Disc) and full 3D simulations (3D-Disc) are compared with the volume averaging technique (VAT) in Figure 7. The results are presented for a volume ratio $\Omega_c = 0.283$, $G_{ur,ref} = 50$ MPa and a variation of loading conditions with $\Delta\sigma$ = 10 kPa and 50 kPa.

Local equilibrium is perfectly achieved for $\Delta\sigma$ = 10 kPa. However, higher loads result in slightly over predicted radial stresses within the columns whereas the stresses in the soil and VAT are almost equal.

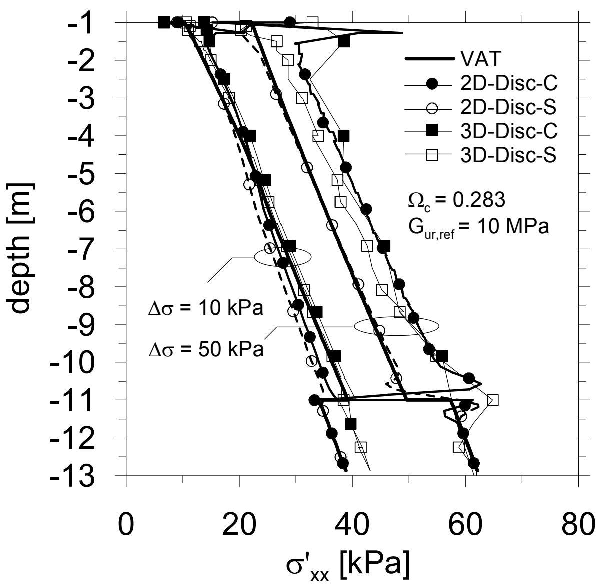

Figure 7. Radial stresses in column, soil and VAT.

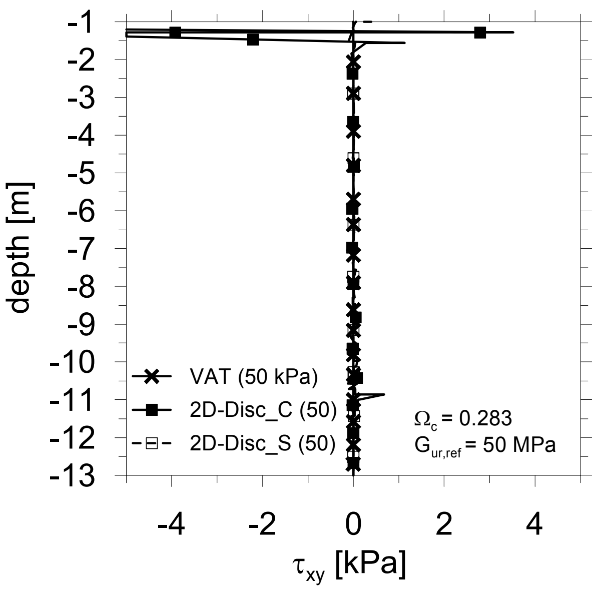

Figure 8. Shear stresses in column, soil and VAT.

For shear stresses local equilibrium is achieved for all loading conditions, variations of input parameters and volume ratios. Figure 8 shows for example the results for higher loads. Scatter at the top and toe of column is due to the load transfer and is negligible.

The radial strains have to achieve equilibrium for radial stresses between column and soil. It can be seen from Figure 9 that radial strains of the virtual column within VAT are in accordance with the discrete simulations. But the radial strains of VAT are increasing with higher load or reduced volume ratio.

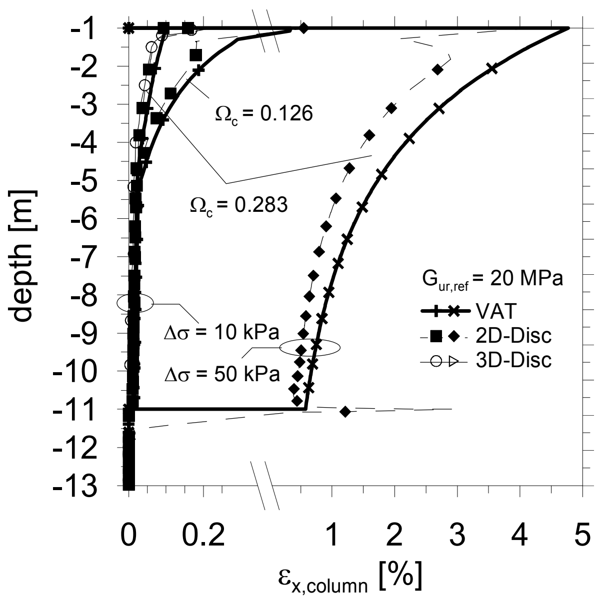

Figure 9. Radial strains in column and VAT.

4.3 Kinematic conditions

The kinematic conditions of the volume averaging technique are based on perfect bonding between column and soft soil, i.e. no differential displacements between column and soft soil.

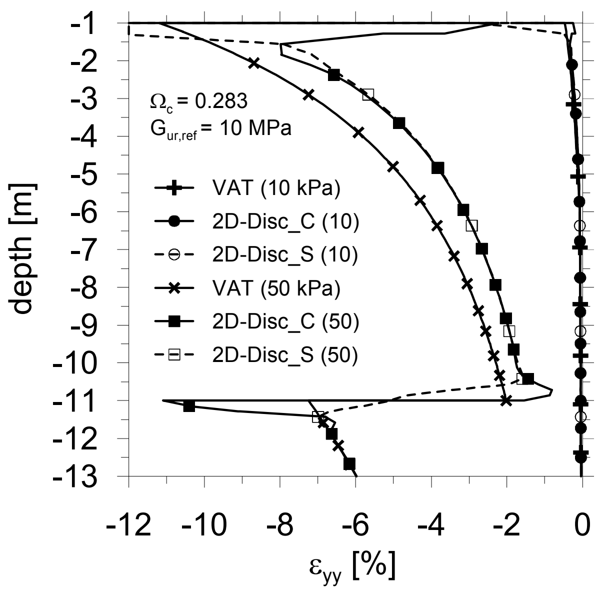

Figure 10. Axial strains in column, soil and VAT.

Perfect bonding is achieved for the presented variations. Figure 10 shows the axial strains with a perfect match for 10 kPa loading conditions. There is a small overestimation of axial strains of VAT with increasing vertical load which is still acceptable. Whereas the vertical strains in column and soil are showing perfect bonding with the discrete simulation of floating columns.

4.4 Settlements

The time dependent settlements in the centre of the column and the soil at the unit cell boundary are shown in Figure 11 for 10 kPa loading and in Figure 12 for 50 kPa loading with variations of column stiffness.

The previous described sensitivity of the volume averaging technique related to increasing loading conditions and their influences on the equilibrium and kinematic conditions can be seen in the settlements as well. The VAT provides very good results for the 10 kPa load case and slightly overestimates the settlements for 50 kPa loading.

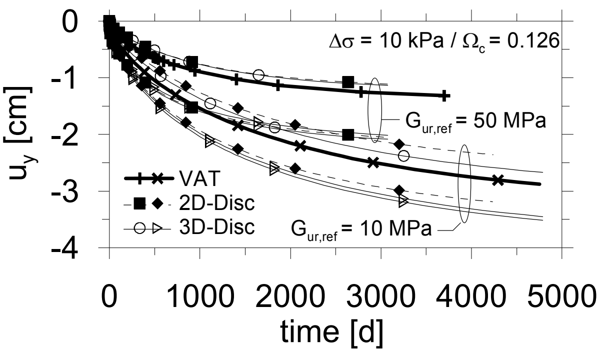

Figure 11. Settlements (load $\Delta\sigma = 10$ kPA).

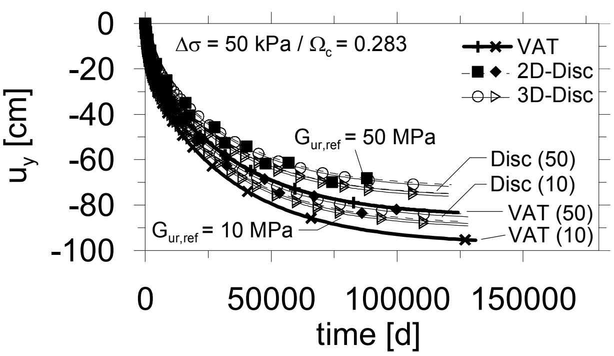

Figure 12. Settlements (load $\Delta\sigma = 50$ kPA).

5 CONCLUSIONS

The performance and accuracy of the volume averaging technique has been demonstrated with simulations of floating columns in a unit cell. The technique has been verified against conventional, fully three-dimensional analyses, where soft soil and deep mixed columns were modelled separately.

The volume averaging technique provides a good agreement with conventional and full 3D analyses. Equilibrium and kinematic conditions have been shown to satisfy the requirements within the natural range of applicability of ground improvements with deep mixed columns in soft soil.

Furthermore, the volume averaging technique allows for fully coupled analyses. The implementation allows for the determination of the time-settlement behaviour of improved areas.

6 ACKNOWLEDGEMENTS

The research leading to these results has received funding from the European Community's Seventh Framework Programme FP7/2007-2013 under grant agreement n° PIAG-GA-2009-230638.

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