Prediction on the long-term behavior of subsoils under high-speed railways

Prédiction du comportement à long terme de sols sous des chemins de fers à grande vitesse

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ABSTRACT: In this paper, some in-situ measurements related to the dynamic stress of the substructure and the subsoil of railway traffic are illustrated. By analyzing the test results from literature, some empirical stress-strain-cycle number relationships of ballast and sands are formulated. Based on these, a quasi-static model predicting the long-term deformation and the stability of the railway foundation is proposed and implemented by using the finite element method. To illustrate the applicability of this model, a calculation example is given.

RÉSUMÉ: Dans ce papier, sont illustrées quelues mesures in-situ en relation avec la sollicitation dynamique des sous-structures et sols du trafic de chemins de fers. Par l'analyse des résultats de tests, recherchés dans la littérature, sont formulés quelques nombres empiriques de cycles de sollicitations en relation avec du lest (ballast) et sable. Basé sur cela, un modèle quasie-statique prédisant la déformation à long terme et la stabilité de la fondation du chemin de fer est proposé et accomplit par la méthode des éléments finis. Pour illustrer l'application de ce modèle, un exemple de calcul est donné.

#### 1 INTRODUCTION

Railway tracks must be laid on the ground in some way and anchored to it. The dynamic loading from railway traffic is carried by the superstructure, substructure and subsoil. The resulting settlement may lead to the deterioration of track systems. This problem is serious on high-speed lines and especially serious on lines with mixed traffic. Here the resulting dynamic loadings are large, but the allowable total settlement, as well as the differential settlement of the tracks is less than on normal lines. Therefore, the prediction of settlement under such conditions is of practical significance in evaluating the long-term behavior of track system.

In literature two different procedures exist describing the constitutive relationships of soils under cyclic loading. The first is built on the classical theory of anisotropic elasto-plasticity and seems to be theoretically well-founded. This is represented by the nested yield surface model of Mroz et al (1978) as well as by the bounding surface model of Dafalias & Herrmann (1982). It is well known from the test results, that plastic deformation of soils occurs, even if the cyclic loading lies within the static yielding surface. This is put down to the translation of the yielding surface during cyclic loading, similar as the experimentally observed Bauschinger effect of metal on cyclic loading. The formulations mentioned above combine the isotropic and kinematic hardening together and make the consideration of this phenomenon possible. The comparison between theoretical prediction and experimental results under triaxial conditions shows that some aspects of soil behavior under cyclic loading can be predicted qualitatively by using these models, for example the movement of effective stress path toward critical state line during undrained cyclic loading and the existence of critical stress ratio under undrained condition. However, some important aspects espe-

cially the quantitative description of them, such as the plastic volume change as well as the hysteresis phenomenon, may not be properly simulated using these models. Furthermore, using these formulations the cyclic course must be followed step by step and formidable data be stored during the calculation. For practical problems, e.g. railway track, the cyclic number to be analyzed can be more than 10<sup>5</sup>. This prevents them from being used in practice. Although some improvements on the simplification were made, the practical application of such models has been rarely reported.

Another procedure describes the soil behavior directly according to the test results of cyclic loading, and may be more reliable, e.g. Andersen (1991), as well as Song (1990). Here, the computational procedure does not trace each cyclic course step by step. The accumulated plastic deformation during cyclic loading is taken into consideration using the empiric relationships obtained from the tests on representative soil samples. As a result, the calculation procedure is significantly simplified.

In this study, the dynamic stresses of the railway underground are evaluated by analyzing the data from some measuring projects. Based on the analysis of the cyclic triaxial test results of ballast and sands in literature, some empiric relationships are formulated describing the mechanical behavior of such two soils under cyclic loading, especially the long-term behavior. Upon these and some assumptions, a quasi-static model is proposed and then numerically implemented by using the finite element method. A computational example demonstrates the capacity of the model as well as its applicability in practice.

#### 2 MEASURED DYNAMIC STRESS

The dynamic stresses resulting from railway traffic depend on train speed, type of superstructure, depth as well as type of soil. In Figure 1a, the measured dependency of the maximum vertical dynamic stress on train speed is presented for the measuring cross sections 1 and 2 on the Hannover-Würzburg railway line. A clear increase of the resulting dynamic stress in the substructure and the subsoil can be observed within the range of the train speed between 150 and 300 km/h.

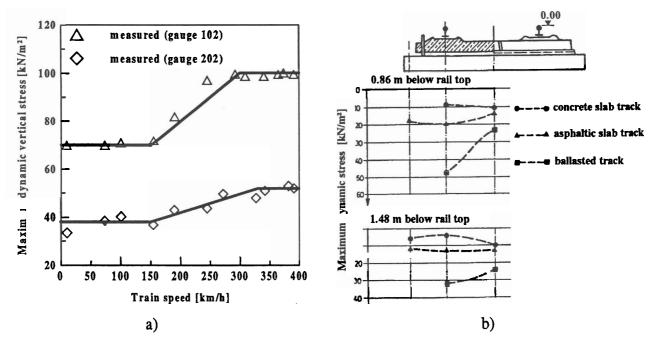


Figure 1. Measured dynamic stress: a) maximum vertical stress versus train speed (project Hannover-Würzburg); b) influence of superstructure (project Kutzenhausen).

The influence of the superstructure on the resulting dynamic stress in the substructure and the subsoil is revealed by analyzing the measured results of the project Kutzenhausen, see Figure 1b.

From ballasted to asphaltic as well as concrete slab tracks, the resulting maximum dynamic vertical stresses become smaller.

Based on the results illustrated above and other analyses, the dynamic loading in the substructure and the subsoil arising from the railway traffic could be evaluated as follows: With reference to the static stress  $\sigma_s$  determined from the wheel set loads, the dynamic increasing factor  $\phi = \sigma_d/\sigma_s$  is 1 up to a train speed of 150 km/h. Beyond that point, the factor increases linearly with train speed until it reaches a maximum at 300 km/h; then,  $\phi$  becomes independent of the train speed again. Based on our analysis of the existing in-situ measurements, the maximum factor  $\phi$  may be about 1.3 for the substructure and the subsoil under slab tracks, and 1.7 under ballasted tracks.

#### 3 STRESS-STRAIN-CYCLIC NUMBER RELATIONSHIP OF GRANULAR SOILS

Extensive experimental studies on ballast and sands have been made under cyclic triaxial condition, e.g. Raymond & Williams (1978), Diyaljee & Raymond (1982), Hettler (1987). The most tests have been carried out with dry materials and low frequency. The influence of pore water pressure as well as frequency are generally not included. Some in-situ measurements indicate that during train passing the resulting excess pore water pressure in granular soils is very low and decreases within very short time down to zero after train passing. Upon this, the factor of pore water pressure may be excluded.

One essential result is that there exists some limit of cyclic deviatoric stress, defined as a ratio to the static limit value  $K = (\sigma_1 - \sigma_3)_{c,f} / (\sigma_1 - \sigma_3)_{s,f}$ . Is the cyclic stress smaller than the limit, the resulting cyclic and permanent deformation of the soil specimens will gradually converge to a corresponding stable value. In this state the soil specimen behaves as a quasi-elastic material. In contrast to this, the cyclic and plastic deformation increases nearly linearly from cycle to cycle leading to failure within short time, if the limit is exceeded.

For the cyclic stable case, a lot of empiric relationships describing permanent vertical strain  $\varepsilon_a^{cp}$  under triaxial condition have been proposed in literature. According to the reanalysis by the authors, the semi-logarithmic and double-logarithmic functions may be applied for ballast and sands, respectively:

ballast: 
$$\epsilon_a^{cp} = a \cdot (1 + \alpha \cdot log N)$$

sand: 
$$\log \varepsilon_a^{cp} = \log a + \alpha \cdot \log N$$

The analysis shows that the coefficient  $\alpha$  is nearly independent of cyclic deviatoric stress ( $\sigma_1$  -  $\sigma_3$ )<sub>c</sub> as well as static hydrostatic stress  $\sigma_3$ , and therefore can be seen as constant with a given relative density. For the coefficient a, the following empiric relationships have been proposed:

ballast: 
$$a = \beta \cdot (\sigma_3 / P_a)^{\alpha} \cdot q^2$$
 (  $P_a = 100 \text{ kN/m}^2$ )

sand: 
$$a = \beta \cdot (P_a / \sigma_3)^{1/3} \cdot q^{\chi}$$
 ( $\sigma_3 < 35 \text{ kN/m}^2$ );  $a = \beta \cdot (\sigma_3 / P_a)^{\chi} \cdot q^2$  ( $\sigma_3 > 35 \text{ kN/m}^2$ )

Here, the cyclic stress level is defined as  $q = (\sigma_1 - \sigma_3)_c / (\sigma_1 - \sigma_3)_{s,f}$ .  $\beta$  and  $\chi$  are two curve fitting coefficients.

Another important relationship for modeling the long-term deformation is the permanent radial strain  $\varepsilon_r^{cp}$  under triaxial condition. Unfortunately, there exist only few available results from literature. The reanalysis of the test results of the ballast materials from Raymond & Williams (1978) indicates that the ratio  $\varepsilon_r^{cp}$  / $\varepsilon_a^{cp}$  may be expressed as a function f(N) as follows:

$$\varepsilon_r^{cp}/\varepsilon_a^{cp} = f(N) = \lambda - \omega \cdot log N$$

 $\lambda$  and  $\omega$  are two curve fitting coefficients. They are dependent on q and  $\sigma_3$  and may be assessed by using the following empiric relationships:

$$\lambda = \lambda_1 \cdot (1 - \sigma_3/\sigma_{3,0}) + \lambda_2 \cdot q$$
 and  $\omega = \omega_0 \cdot (\sigma_3/P_a)^{1/2}$ 

 $\lambda_1, \lambda_2, \sigma_{3,0}$  and  $\omega_0$  are determined from test results.

### 4 CALCULATION MODEL

For simulating the stress-strain-relationship of ballast and sands observed in cyclic triaxial tests, a quasi-static model is proposed, see Figure 2a. Here, the maximum of cyclic dynamic loading is applied as a quasi-static stress  $\sigma$  in the system. The model consists of 4 elements: spring E, nonlinear viscous dashpot  $\eta_1$ , linear viscous dashpot  $\eta_2$  and limiting value  $\sigma_k$ .

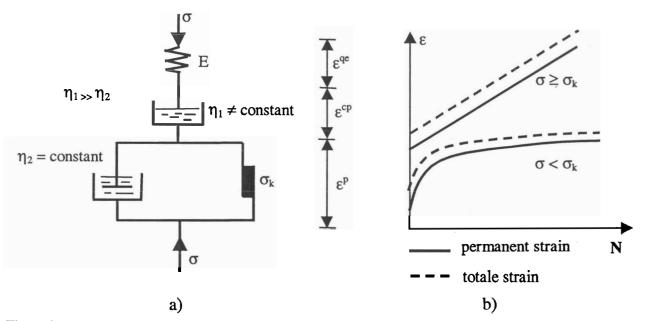


Figure 2. a) One-dimensional quasi-static model;

b) Schematic strain - cycle number - relationship.

According to the results from literature, the cyclic strain part  $\epsilon^{qe}$  is generally much smaller than the permanent strain  $\epsilon^{cp}$ . Therefore, it is rational to assume that the quasi-elastic modul E is independent of the cycle number. The viscous dashpot  $\eta_1$  is introduced to simulate permanent strain  $\epsilon^{cp}$  for the cyclic stable case and is dependent on the cyclic number N. This dependency can be determined using the empiric relationship proposed in section 3. For the failure case, the conventional visco-plastic formulation is used, see Figure 2. The limiting value  $\sigma_k$  can be calculated using the parameter K and the static strength parameter  $\varphi$ .

The one-dimensional conceptual model has been generalized to the three-dimensional case assuming that the principle of superposition can be applied to compute the permanent strain resulting from cyclic stress components in the three primary directions. This generalized quasi-static model has been implemented in the FE-Program "GEOCYCL" by using the initial strain algorithm.

### 5 A CALCULATION EXAMPLE

In Figure 3a, a conventional ballasted track on the uniform fine-grained sand is illustrated as a calculation example. It is assumed that the wheel load of a train can be idealized as a line load with a value of 30 kN/m. The design speed is assumed to be 300 km/h and the cycle number up to 10<sup>5</sup>. Under consideration of the dynamic increasing factor, described in section 2, the quasi-static line load should be 1.7×30 kN/m. The chosen cross-section and the two-dimensional FE-mesh is shown

in Figure 3b. The essential parameters used in the calculation are given in Figure 3. The numerical calculation was carried out on PC (Pentium II) and the total calculation time was about 30 minutes.

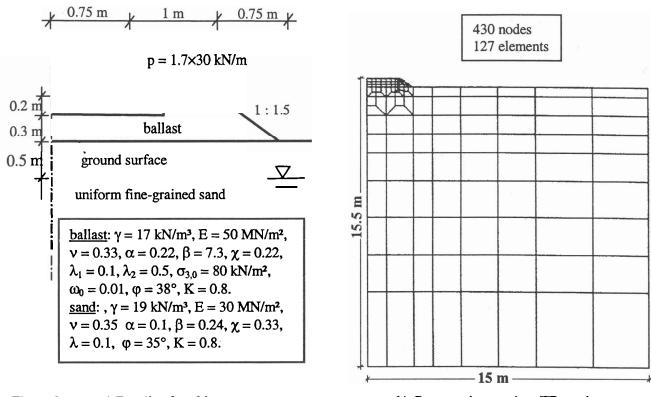


Figure 3. a) Details of problem;

b) Computation section, FE-mesh.

The calculated total settlement of sleeper and ground surface depending on the cycle number is illustrated in Figure 4a. A total settlement of ca. 13.3 mm is predicted at a cycle number of 10<sup>5</sup>. The difference between the two curves shows the deformation part resulting from the ballast. The deformation rate decreases quickly in the initial 20.000 cycles and then becomes slowly. In Figure 4b, the distributions of the settlement on the ground surface are shown for some chosen cycle numbers. The plastic zone in ballast and underground depending on the cycle number is illustrated in Figure 5. No further extension of the plastic zone can be observed after the cycle number of 500.

# 6 CONCLUSIONS

The in-situ measurements indicate that for train speed between 150 and about 300 km/h the resulting maximum dynamic stress in the substructure and subsoil exhibits large dependency on the train speed. In addition, the type of superstructure has large influence on the resulting dynamic stress. The measured stress under the conventional ballasted tracks is much larger than those under the slab tracks.

Based on the reanalysis of the existing results of triaxial tests from literature, some empiric stress-strain-cycle number-relationships have been formulated for ballast and sands. For simplifying the calculation procedure, a quasi-static formulation has been proposed. The corresponding three-dimensional formulation has been implemented in a finite element model.

A two-dimensional example of the railway foundation on uniform fine-grained sand has been calculated by using the developed program "GEOCYCL". The predicted settlement and differential settlement as well as the plastic zone are illustrated as a function of cycle number. The calculation results seem to be feasible and the proposed model applicable for practice.

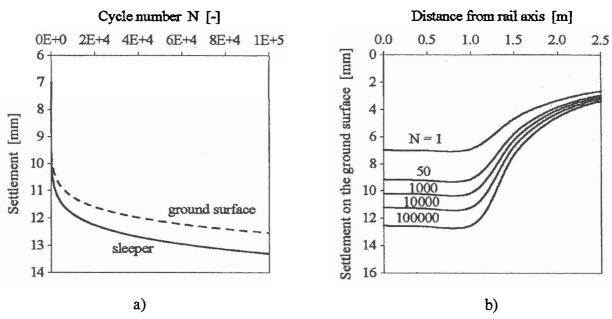


Figure 4. a) Settlement depending on cycle number; b) Settlement distributions on the ground surface.

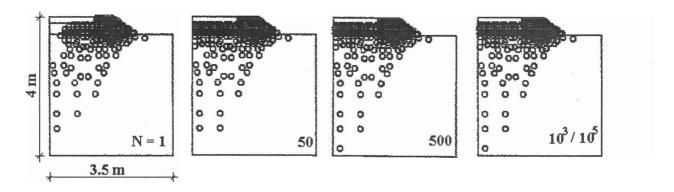


Figure 5. Development of plastic zone.

## **REFERENCES**

Andersen, K. H. 1991. Foundation design of offshore gravity structures. In O'Reilly et al (eds.), Cyclic loaing of soils,

Dafalias, Y. F. & Herrmann, L. R. 1982. Bounding surface formulation of soil plasticity. In Pande & Zienkiewicz (eds.), Soil Mechanics-Transient and cyclic loads. Wiley, London.

Diyaljee, V. A. & Raymond, G. P. 1982. Repetitive load deformation of cohesionless soil. J.Geotech. Engrg., ASCE, 108(10).

Hettler, A. 1987. Schottertriaxialversuche mit statischem und zyklischem Belastungsverlauf. ETR 36.

Mroz, Z., Norris, V. A., & Zienkiewicz, O. C. 1978. An anisotropic harding model for soils and its application to cyclic loading. Int. J. Num. Ana. Methods in Geomech. Vol. 2.

Raymond, G. P. & Williams, D. R. 1978. Triaxial tests on dolomite railroad ballast. J. Geotech. Engrg., ASCE, 104(6).

Song, E. X. 1990. Elasto-plastic consolidation under steady and cyclic loads. Research report of the Technical University Delft.