Calibration of soil parameters for an elasto-plastic-cap model under drained conditions

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ABSTRACT: The paper presents a study of the performance of the hardening soil model (HSM) under drained triaxial condition. In addition to the parameters from triaxial test to control the plastic strains that are associated with the shear yield surface, the HSM requires parameters from oedometer test such as the constrained modulus to define the plastic strains that originate from the yield of the cap. Thus, the validation of the model is carried out parallel for both type of loading conditions. Moreover, the influence of each model parameters on the stress–strain behaviour and the volume change characteristic has been studied and presented. Finally, a summary of the influence of the various parameters is presented in a matrix form.

1 Introduction

There are three terms often mentioned and discussed nowadays in computational mechanics. These are verification, validation and calibration. Verification is defined as the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model (code-to-analytical solution comparison). Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model (code-to-experimental data comparisons). Calibration is the process of adjusting physical or numerical modelling parameters, components or aspects of the computational model for the purpose of implementing a computational model or improving agreement with the experimental data (material model parameter determination) (Roache, 1998; Oberkampf et al., 2002). In this paper, an attempt is made to validate and calibrate the hardening soil model. Moreover, a sensitivity study is carried out to examine the influence of the model parameters on the stress-strain, strength and volume change characteristics. In addition to the parameters from triaxial test to control the plastic strains that are associated with the shear yield surface, the hardening soil model requires parameters from oedometer test such as the constrained modulus to define the plastic strains that originate from the yield of the cap. Thus, the validation of the model is carried out parallel for both type of loading conditions.

2 The constitutive soil model

The constitutive model to which the soil parameters are being calibrated in this paper is an elasto-plastic-cap soil model known as the hardening soil model (HSM). The HSM is implemented in the finite element code for soils and rocks "PLAXIS" (Brinkgreve, 2002). It is originally developed based on the so called the Duncan-Chang hyperbolic model. It, however, supersedes the hyperbolic model, because it uses the plasticity theory instead of the elasticity theory, it includes the dialatancy soil behaviour and it introduces the yield cap. The HSM also considers the stress dependant stiffness of the soil according to the power law. The basic features of the HSM are listed in Table 1. For more and detail Information on the constitutive model and the program PLAXIS refer to Brinkgreve (2002).

Table 1. Basic features of the HSM

Type of model:	• elasto-plastic strain hardening cap model	State of stress:	• isotropic
Basic features:	• stress dependent stiffness according to power law • $E = E^{ref}\left(\frac{c' \cdot \cos\varphi - \sigma_3' \cdot \sin\varphi}{c' \cdot \cos\varphi + p^{ref} \cdot \sin\varphi}\right)^m$ • plastic straining due to primary deviatoric loading • plastic straining due to primary compression • elastic unloading/ reloading • hyperbolic stress-strain relation • soil dilatancy	Cap yield surface:	$F_c = \frac{\bar{q}^2}{\alpha^2} + p^2 - p_p^2$, $\bar{q} = \sigma_1 + (\delta - 1) \cdot \sigma_2 - \delta \cdot \sigma_3$ • α is a model parameter that relates to K_0 • p_p is an isotropic pre-consolidation stress • p is the effective mean stress • $\delta = \frac{3 + \sin\varphi}{3 - \sin\varphi}$
		Hardening:	• isotropic; shear and compression
Failure criterion:	• Mohr-coulomb	Required soil parameters:	• c', φ', ψ, E_{50}^{ref}, E_{ur}^{ref}, E_{oed}^{ref}, m, K_0^{nc}, ν_{ur}
Flow rule:	• non-associated in shear hardening • associated in compression hardening (cap)	Range of applications:	• all types of soils

3 The finite element model

The triaxial test and the oedometer test are simulated by means of an axisymmetric geometry, with the real dimension of the test set-up, that represent half of the soil sample (0.025 x 0.05 m in case of the triaxial test and 0.035 x 0,02 m in case of oedometer test). In the triaxial model, the displacements normal to the boundaries are fixed and the tangential displacements are kept free to allow for smooth movements along the axis of symmetry (the left hand side) and the bottom boundaries. The top and the right side boundaries are fully free to move. Similarly, the displacements normal to the boundaries are fixed and the tangential displacements are kept free to allow for smooth movements along the axis of symmetry (the left hand side) and the right hand side boundaries in the oedometer model. Both the normal and tangential displacements along the bottom boundary are fixed, whereas the top boundary is fully free to move.

4 Validation of the Hardening Soil Model

4.1 The soil parameters

The soil parameters required for HSM are given in Table 2. They are obtained from extensive drained triaxial and oedometer test results conducted on undisturbed specimen of lacustrine soft soil (Gebreselassie, 2003). They are mean values of several tests and they will serve as reference parameters in the following numerical computations.

Table 2. Reference soil parameters

γ_{sat} [kN/m³]	φ' [°]	c' [kN/m²]	E_{50}^{ref} [kN/m²]	E_{oed}^{ref} [kN/m²]	E_{ur}^{ref} [kN/m²]	p^{ref} [kN/m²]	m [-]	R_f [-]	K_0^{nc} [-]	ν_{ur} [-]
19.5	25.3	13.2	3253	2948	19170	100	0.63	0.83	0.573	0.20

4.2 The calculation

The triaxial test procedure is modelled by means of applying first an all round confining pressure $\sigma_3' = 50$, 100 and 200 kN/m² for three specimens respectively and then by increasing the vertical stress by $\Delta\sigma$ up to failure. The choice of the three confining pressure makes possible the study of the influence of the different soil parameters at the reference pressure p^{ref}=100 kN/m² and at stress level below and above the reference pressure. Similar to the test condition, the following load increments are used in the FEM - simulation of the oedometer test: 10.8, 20.1, 30, 69, 126, 252, 126, 69, 10.8, 69, 126, 252, 504, 756, 504, 126, 12.6 kN/m².
In order to study the effect of the different hardening soil model parameters on the stress-strain, the strength and the volume-change behaviour of the soil specimens, several variations of the soil parameters have been considered during the FEM-computations. These variations are listed in Table 3. The reference soil parameters are adopted from Table 2.

Table 3. Variations of the HSM parameters

Case	Parameter variation	Case	Parameter variation
FEM-1	reference parameters (Table 2))	FEM-11	= FEM-1, but $E_{ur}^{ref} = 3 \cdot E_{ur}^{ref}$ = 9759 kN/m²
FEM-2	= FEM-1, but E_{50}^{ref} increased by a factor of 1.25	FEM-12	= FEM-1, but K_0^{nc} increased to 0.71
FEM-4	= FEM-1, but E_{50}^{ref} increased by a factor of 2.0	FEM-13	= FEM-1, but K_0^{nc} reduced to 0.48
FEM-6	= FEM-1, but $m = 0.83$ (from oedometer test)	FEM-14	= FEM-1, but $\nu_{ur} = 0.10$
FEM-8	= FEM-1, but $E_{oed}^{ref} = E_{50}^{ref}$ = 3253 kN/m²	FEM-15	= FEM-1, but $\nu_{ur} = 0.30$
FEM-9	= FEM-1, but E_{oed}^{ref} reduced by a factor of 0.75	FEM-16	= FEM-1, but $R_f = 0.97$
FEM-10	= FEM-1, but E_{oed}^{ref} increased by a factor of 1.25	FEM-17	= FEM-1, but $R_f = 0.67$

4.3 Analysis of the computation results

4.3.1 Stress-strain behaviour

The stress strain relationship of the soil specimen from the FEM computation and test results are presented in Figure 1. The test results are indicated with dashed lines and the shaded regions show the range of the variation of the test results. Since the hardening soil model requires soil parameters both from the triaxial test and one-dimensional compression test, the comparison of the FEM- results are presented parallel, for example, Figure 1a for the triaxial loading system and Figure 1b for oedometer loading condition.
It would appear from Figure 1a that the computational results of the triaxial model for the reference case (FEM-1) underestimate the stiffness of the soil specimen at an axial strain less than 5 - 6 % for all cases of confining pressures. This might happen due to the fact that the hardening soil model use the secant modulus E_{50} instead of the initial tangent modulus E_i ($E_i \approx 2 \cdot E_{50}$, see

Gebreselassie, 2003). Such problem may be overcome by introducing two hyperbola with two different stiffness lines (Amann et al., 1975), and loading the specimen piecewise in two steps each with different material sets (see also Gebreselassie, 2003). The FEM-simulation of the triaxial test, however, lies reasonably within the range of variations of the test results for an axial strain greater than 5 - 6%.

On the contrary to the triaxial simulation, the FEM-simulation of the oedometer (FEM-1) overestimates the stiffness of the specimen up to a vertical strain of 7% , and thereafter it joins the region of the range of the test results (Figure 1b).

The FEM simulates very well the un/reloading stiffness of the specimen in the triaxial loading condition, whereas it overestimates it in the oedometer loading condition. Lowering the un/reloading modulus to $E_{ur}^{ref} = 3 \cdot E_{50}^{ref}$ (FEM-11), which is given in PLAXIS as a default value, would result in underestimation of the un/reloading stiffness of the triaxial test, whereas it still overestimates it in the oedometer test for the 1st un/reloading at about a vertical pressure of 200 kN/m², but much closer to the test result than the reference case (FEM-1). For the 2nd un/reloading case at higher stress level in the oedometer test, the FEM simulation underestimates the un/reloading stiffness. If one wants to keep the triaxial un/reloading stiffness unchanged, since it match very well to the test results, and on the other hand to adjust it to the test results in the oedometer simulation, the only possibility available is to vary the value of the Poisson's ratio for un/reloading ν_{ur}. This is the only parameter that influences the un/reloading behaviour of the one-dimensional compression without affecting much the un/reloading behaviour in the deviatoric state of stress.

Apart from its influence on the un/reloading behaviour of both test conditions, E_{ur}^{ref} has no significant influence on the stress-strain behaviour of the specimen during the 1st loading.

The assumption $E_{oed}^{ref} = E_{50}^{ref}$ (FEM-8), which is recommended in PLAXIS as a default value, has no significant influence on the deviatoric stress-strain behaviour, whereas it reacts stiffer in one-dimensional compression (Figure 1). This approves that E_{50}^{ref} is largely a compression hardening parameter (cap parameter). In both loading systems, the assumption $E_{oed}^{ref} = E_{50}^{ref}$ has no effect on the un/reloading stress -strain behaviour.

Taking the value of the exponent $m = 0.83$ (FEM-6) from oedometer test result instead of $m = 0.63$ from triaxial test increases the stiffness of the soil specimen for a stress level above the reference pressure and decreases the stiffness for a stress level below the reference pressure in both loading conditions (Figure 1) as expected. The optimal solution seems to lay between these values.

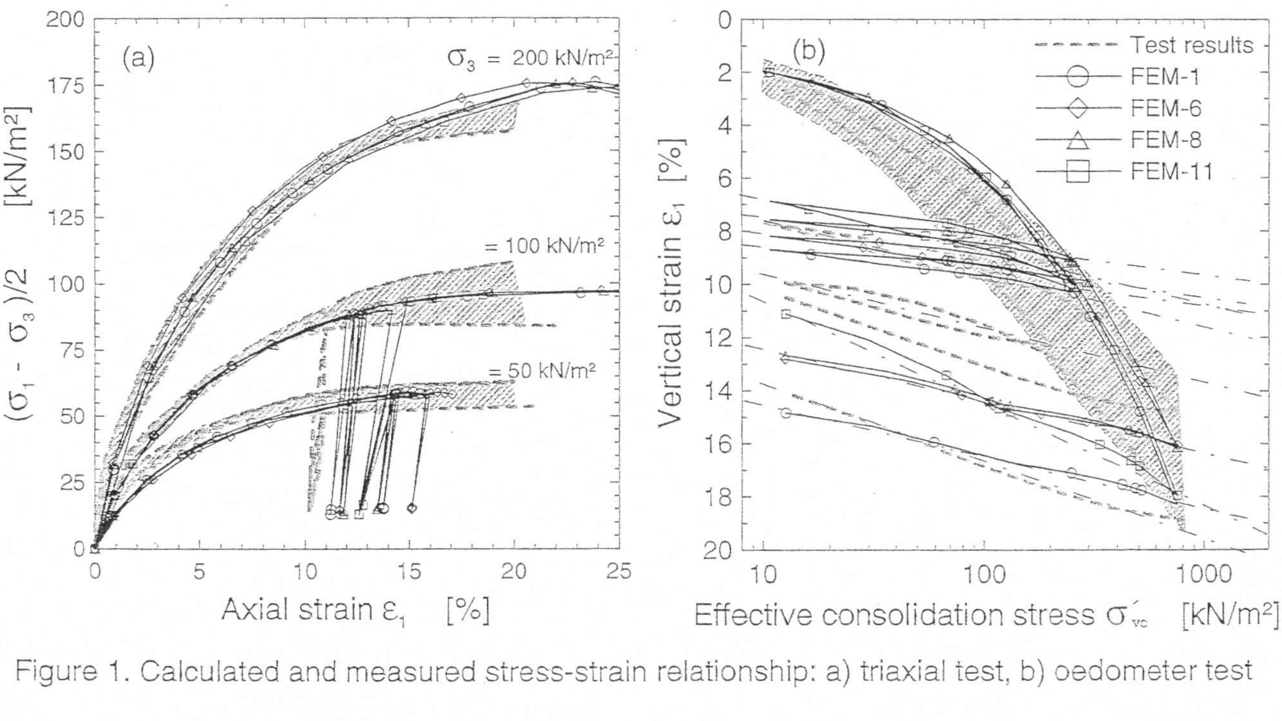

Figure 1. Calculated and measured stress-strain relationship: a) triaxial test, b) oedometer test

It would appear from Figure 1a that the deviatoric stress at failure remains unaffected by the variations of the parameters m , E_{oed}^{ref} and E_{ur}^{ref} , although the strain at which the failure occur might be different. This is because the failure stress is mainly controlled by the shear parameters c' and φ' in the drained analysis.

4.3.1.1 Influence of the stiffness parameters E_{50}^{ref} , E_{oed}^{ref} and E_{ur}^{ref} on stress-strain behavior:

In order to study the influence of the different hard soil model parameters on the stress - strain behavior of the soil specimen, various FEM - computations are conducted according to the cases listed in Table 2. The first group of variations are the stiffness parameters E_{50}^{ref} , E_{oed}^{ref} and E_{ur}^{ref} . Increasing the value of E_{50}^{ref} by 25% (FEM-2) shifts the reference curve upwards and partly lies above the range of the measured values, but it joins the reference curve as it approaches failure (Figure 2a). E_{50}^{ref} has no effect at all on the one-dimensional compression as shown in Figure 2b. On the other hand, changing the value of E_{oed}^{ref} by ±25% (FEM-9 & 10) has no significant influence on the deviatoric stress, whereas it affects the stress -strain characteristics of the one-dimensional compression accordingly. This is a clear proof of the fact that the parameter E_{50}^{ref} is purely shear hardening parameter (shear yield surface), whereas the parameter E_{oed}^{ref} is purely a compression hardening parameter (cap yield surface).

Although lowering the value of the parameter E_{ur}^{ref} as much as 50% of the reference value (FEM-11) has no significant influence on the stress - strain curves of both loading systems during the first loading, it affects both loading systems equally during un/reloading. Hence, E_{ur}^{ref} is a parameter common to both yield surfaces.

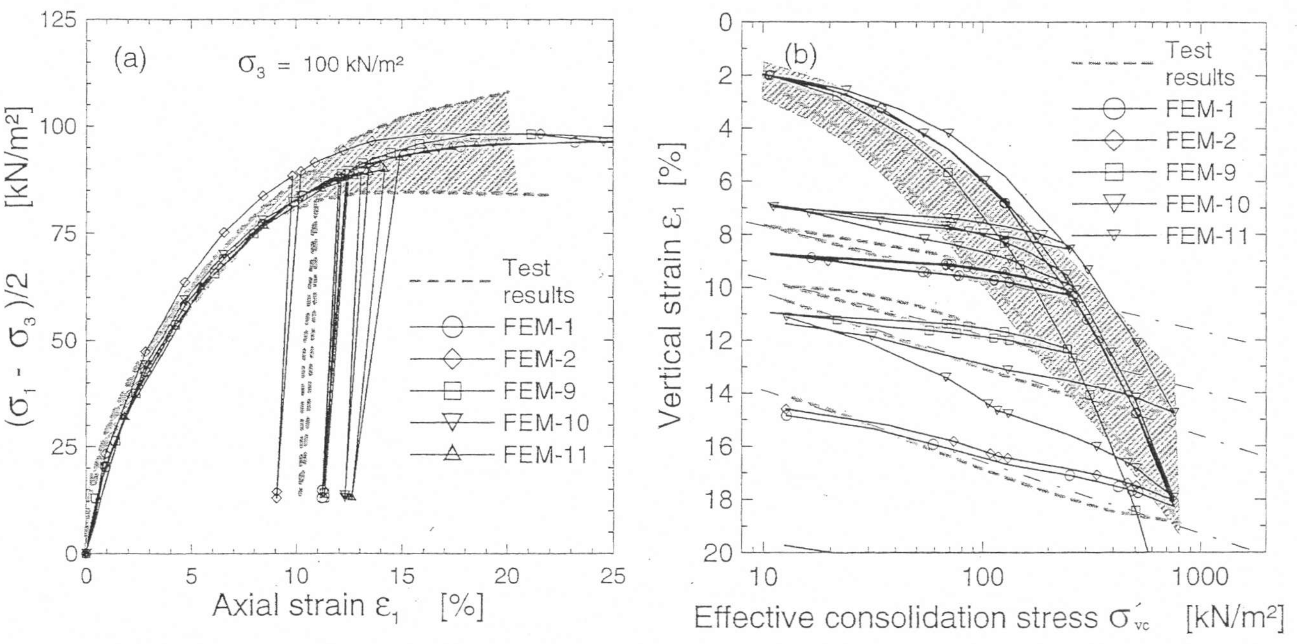

Figure 2. The influence of the stiffness parameters E_{50}^{ref} , E_{oed}^{ref} and E_{ur}^{ref} on the stress-strain characteristics of a soil specimen a) triaxial case ($p^{ref} = 100\, kN/m^2$), and b) oedometer case

4.3.1.2 *Influence of the parameters ν_{ur}, K_0^{nc} and R_f on stress-strain behaviour:*

In the second group of variation belong the parameters ν_{ur}, K_0^{nc} and R_f. Varying the value of ν_{ur} to 0.1 (FEM-14) and to 0.3 (FEM-15) has no significant influence on the deviatoric primary loading and un/reloading state of stress (Figure 3a), whereas it has a considerable effect on the un/reloading stiffness of one dimensional compression (Figure 3b). Whereas lowering ν_{ur} to 0.1 decreases the un/reloading stiffness and fairly approaches the test result, increasing ν_{ur} to 0.3 tends to increase the un/reloading stiffness and diverges further from the test results. ν_{ur} is the single parameter that affects the un/reloading stiffness of the one-dimensional compression without affecting the corresponding un/reloading stiffness of the deviatoric loading system. If a match of the computation and the test results during the un/reloading state is desired, this is the suitable parameter for a variation to deal with.

The HSM distinguishes between the model parameter K_0^{nc} and K_0 which defines the initial state of stresses. Since the initial stresses in the very small triaxial model will have no as such an influence on the stress-strain behaviour, it is assumed that $K_0^{nc} = K_0$. Increasing the value of K_0^{nc} by 25% (FEM-13) results in a divergence of the stress - strain curve below the reference curve whereas decreasing its value by the same amount (FEM-12) leads to an increase of the stiffness of the soil above the reference value in both triaxial (Figure 3a) and oedometer(Figure 3b) loading systems. However, the effect of varying K_0^{nc} seems to be stronger for the triaxial loading system than for the one-dimensional loading system. In both cases, K_0^{nc} seems to have no significant influence on the un/reloding state of stress.

The lines of the FEM-simulation of the variation of the influence of the failure factor R_f above (FEM-16) and below (FEM-17) the reference value in Figure 3a, follows the course of the reference curve up to approximately an axial strain of 5% from which they start to diverge upwards (FEM-17) and downwards (FEM-16). Its final effect is to retard the failure in the case of increasing its value and to accelerate it in the case of lowering its value. However, the influence of increasing the R_f value above the reference value seems to be larger than the opposite one.

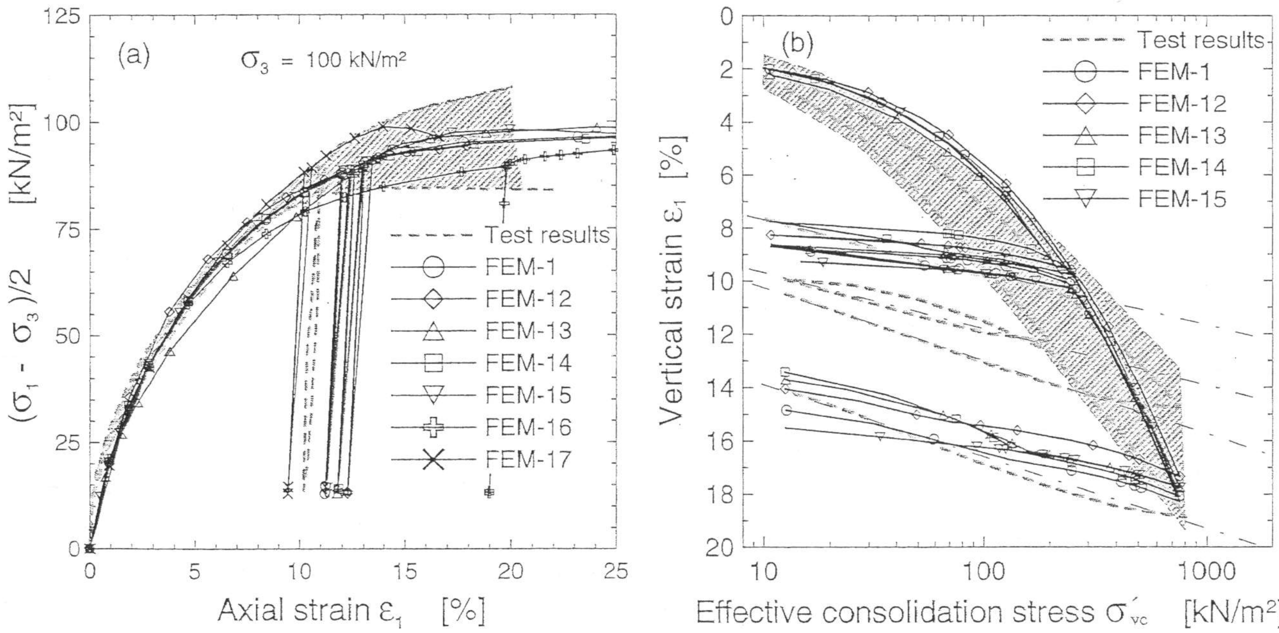

Figure 3. The influence of the parameters ν_{ur}, K_0^{nc} and R_f on the stress-strain characteristics of a soil specimen: a) triaxial case ($p^{ref} = 100\,kN/m^2$), b) oedometer case

4.3.2 *Volume change behaviour*

The volume change behaviour of the specimen under drained triaxial test condition has also been studied by means of varying the soil parameters of the hardening soil model. The results of the element study against the test results for a specimen with a confining pressure of 100 kN/m² are shown in Figure 4. It can be seen from this figure that the range of the test results is very wide and it is difficult to compare the computational result with the test result directly. However, one can see the general tendency of the volume change behaviour from the test results and the influence of each parameter from the sensitivity study. From Figure 4, it would appear that all the parameters in one way or the other way may affect the volume change behaviour of the specimen. The most sensitive parameters with regard to the volume change behaviour, however, are E_{oed}^{ref} and K_0^{nc} (FEM-8, 9 & 10) and (FEM-12 & 13), and the least sensitive parameter is ν_{ur} (FEM-14 & 15). It is interesting to see that increasing E_{50}^{ref} value above the reference value increases the volumetric strain, when one expects the opposite result.

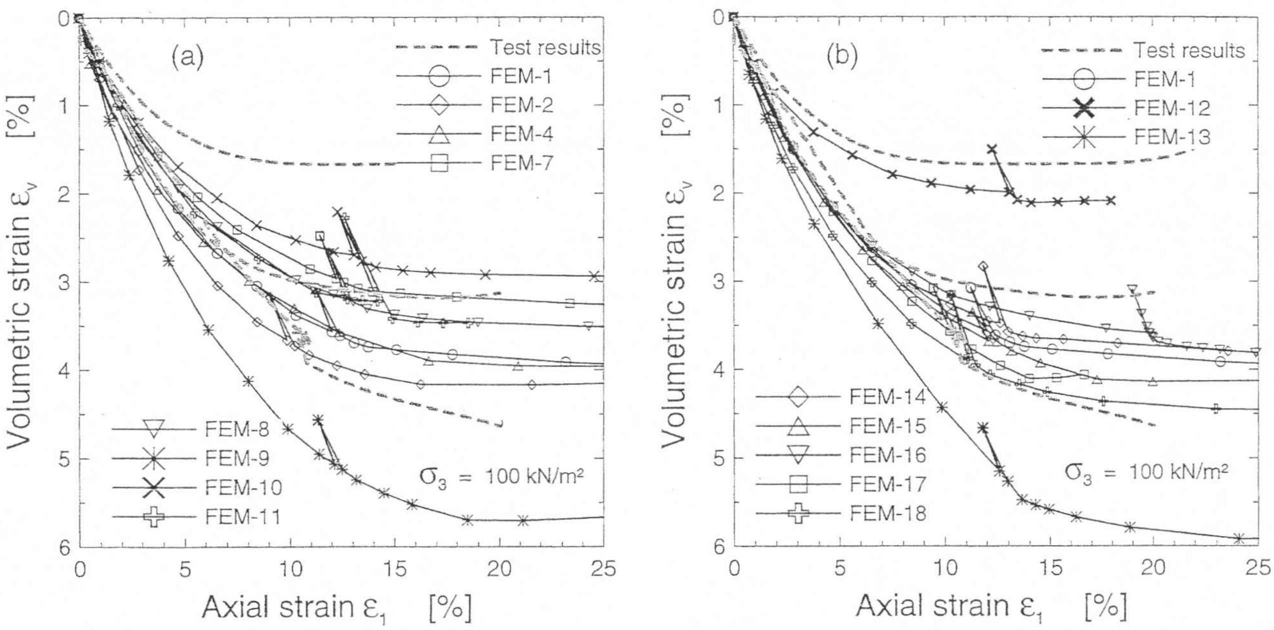

Figure 4. The influence of the HSM parameters on the volume change behavior of a soil specimen at confining pressure of 100 kN/m²: a) stiffness parameters: E_{50}^{ref}, E_{oed}^{ref}, E_{ur}^{ref}, and m, b) other parameters: K_0^{nc}, R_f and ν_{ur}

5 Summary

The drained tests can fairly be simulated with the FEM with a slight modification of parameters. As would expected, E_{50}^{ref} is the main parameter that determines the stress-strain behaviour of a soil specimen in a triaxial primary loading condition, and it has negligible influence on one-dimensional loading condition. Similarly, E_{oed}^{ref} has insignificant effect on the triaxial loading condition, but plays the main role in one-dimensional loading. E_{ur}^{ref} is the single parameter that influences the un/reloading condition both in triaxial and one dimensional state of stresses. Contrary to the expectation, ν_{ur} shows no effect on the one-dimensional un/reloading condition. It seems that K_0^{nc} will have no significant influence on the un/reloading state of stress in both loading systems. In general, it is observed that the FEM-simulation underestimates the stiffness of the soil at a lower strain (say up to around 2.5% axial strain).

E_{oed}^{ref} and K_0^{nc} values appear to play the leading role in determining the volume change characteristic of the specimen in a triaxial compression, although all the other parameters with the exception of R_f contribute their part. This and the above discussion show the separate function of E_{oed}^{ref} as a cap parameter that controls the compression hardening and E_{50}^{ref} as a parameter that controls the shear hardening. The influence of the different hardening soil model parameters on the stress strain, strength and volume change behaviour of a soil specimen keeping the effective shear parameters constant is summarised in Table 4.

Table 3. Summary of the results

Soil parameter	Stress - Strain behaviour				Volume change	strength at limit state
	Triaxial loading condition		One-dimensional compression			
	loading	un/reloading	loading	un/reloading		
E_{50}^{ref}	✓✓✓	✗	✗	✗	✓✓	✗
E_{oed}^{ref}	✗	✗	✓✓✓	✗	✓✓✓	✗
E_{ur}^{ref}	✓	✓✓✓	✗	✓✓✓	✓✓	✗
m	✓✓	✓	✓✓	✓	✓✓	✗
ν_{ur}	✗	✗	✗	✓✓✓	✓	✗
K_0^{nc}	✓✓	✗	✓	✗	✓✓✓	✗
R_f	✓✓	✗	✗	✗	✗	✗

✓✓✓ = has a considerable effect; ✓✓ = has an effect; ✓ = has a slight effect; ✗ = has no effect

6 List of symbols and abbreviations

E_{50}^{ref} = secant modulus at 50% of the failure stress and at effective reference pressure of p^{ref}
E_{oed}^{ref} = constrained modulus at p^{ref}
E_{ur}^{ref} = un/reloading modulus at p^{ref}
E = modulus of elasticity
γ_{sat} = saturated unit weight of soil
φ' = effective angle of internal friction
δ = wall friction
c' = effective cohesion
R_f = ratio of the stress at failure and the ultimate stress
K_0^{nc} = coefficient of the earth pressure at rest for normally consolidated soils
ν_{ur} = Poisson's ratio for un/reloading
ν = Poisson's ratio
m = exponent in the power law
HSM = Hardening Soil Model
MCM = Mohr-Coulomb Model

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