VOLUME AVERAGING TECHNIQUE FOR MODELLING DEFORMATIONS IN SOFT SOIL IMPROVED WITH DEEP MIXED COLUMNS

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Abstract The deformation behaviour of a foundation on deep mixed columns in a natural soft soil is a three-dimensional problem, which has to be considered adequately in numerical modelling. However, simplifications are often made due to the computational costs of fully coupled 3D analyses by either modifying geometry of the problem or material parameters in 2D for plain strain analyses or by modelling the problem as 2D axisymmetric case. The volume averaging technique (VAT) is an enhanced 2D FE technique that enables mapping the 3D problem in two-dimensions, yet modelling the two constituents (column and soft soil) with appropriate different constitutive models within a homogenized material. The performance of the volume averaging technique is demonstrated by considering unit cell simulations of floating and end bearing columns in soft soil. The results with VAT are compared and validated against conventional 2D axisymmetric and fully coupled 3D finite element analyses.

Introduction

The properties of very soft clays, silts and organic soils can be improved by stabilizing agents, such as lime and/or cement that are mixed into the soil in situ by using auger-type mixing tools in situ. Deep mixed columns are extensively used to reduce deformations and to improve the overall stability of embankments and foundations on soft soils. Whilst the most typical applications are embankments, increasingly the method is used under foundations. The first category of applications is very common in Scandinavian countries, whilst the latter application has been pioneered in Poland, as an environmentally friendly (low CO_2) and cost effective alternative for piling. Due to economic and environmental reasons there is an increasing interest in the technique elsewhere in Europe. With deep mixing, the strength and deformation properties of the soft soil can be improved, and the risk of unwanted deformations is substantially reduced.

Numerical methods, such as finite element (FE) analysis, can be used as an alternative to conventional design methods. They are particularly suitable when considering serviceability limit state, i.e. deformation analyses under working loads. FE analyses allow using advanced constitutive models that take account of the complex stress-strain behaviour of natural soil and stabilized columns, respectively. The problems that involve a grid of circular columns under an embankment or a strip footing are fully three-dimensional problems. As 3D analyses are very laborious and expensive, an enhanced 2D technique using the so-called volume averaging technique (VAT) is utilised. The basic idea is to describe the column-improved ground as a homogenized composite material, and using clever mathematics to map the true 3D problem into 2D. Once the constitutive relations of both composites are defined, the response of the column-improved ground can be simulated in 2D subject to arbitrary loading and boundary conditions.

In this paper the volume averaging technique by Vogler & Karstunen (2007, 2009) is used in which two advanced constitutive models for soft soil and deep mixed column were implemented: the S-CLAY1S model (Karstunen et al. 2005) to represent the soft soil and the MNhard model (Benz 2007) to represent the deep mixed columns. The work presented is an extension of the paper by Becker & Karstunen (2013). The performance of the volume averaging technique is demonstrated by comparing the results of discrete 2D axisymmetric and full 3D models of unit cell simulations of floating and end bearing deep mixed columns, i.e. column and soil are modelled separately, with enhanced 2D analyses using the volume averaging technique. The work is an extension of the study presented by Becker & Karstunen (2013).

Volume averaging Technique

Introduction and fundamental assumptions

The volume averaging technique originates from composite modelling and offers an elegant way to model periodic systems as a homogenous material instead of discrete modelling of the columns and natural soil (Fig. 1). The principles adopted by Vogler & Karstunen (2007, 2009), utilised later on by Becker & Karstunen (2013), are based on the initial idea by Schweiger & Pande (1986), which was further refined by Lee & Pande (1998). The formulation has been extended to three dimensions and a new solution routine has been developed to cope with the highly non-linear constitutive models.

Within the volume averaging technique a periodic arrangement of the columns in the natural soil is assumed. Furthermore, perfect bonding is assumed, i.e. no slip between natural soil and columns is allowed. The method allows for adopting any constitutive model to the two constituents involved: natural clay and column-improved soil. The local equilibrium between soil and column, and most importantly the kinematic compatability, as well as the validity of the constitutive relations are satisfied through stress/strain redistribution within a sub-iterating procedure.
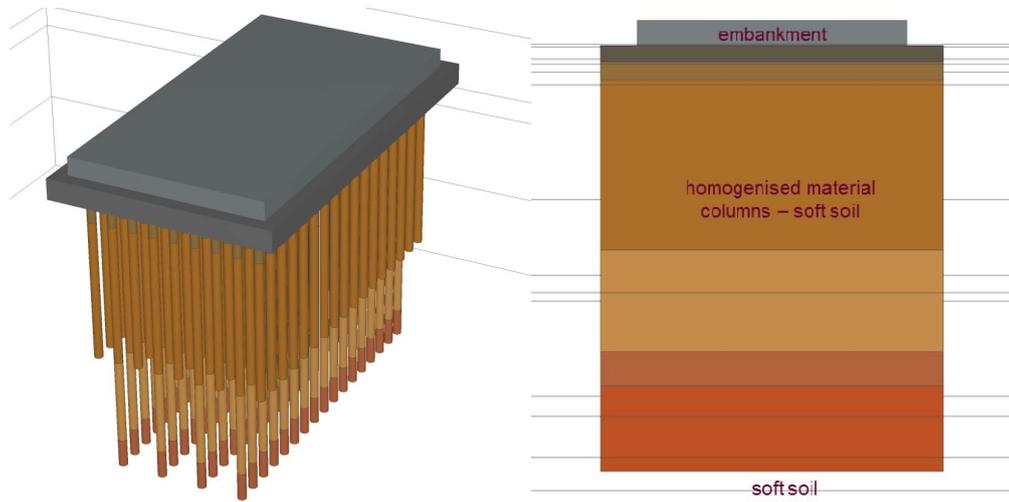

Figure 1. Discrete full 3D simulation (left) and homogenised (right) representation of deep mixed columns in soft soil under an embankment (Becker & Karstunen 2013).

Equivalent material stiffness matrix

The strain increment and the stress increment in the homogenised equivalent material are derived according to the following averaging rules:

$$\dot{\boldsymbol{\sigma}}^{eq} = \Omega_s \dot{\boldsymbol{\sigma}}^s + \Omega_c \dot{\boldsymbol{\sigma}}^c \qquad (1a)$$

$$\dot{\boldsymbol{\varepsilon}}^{eq} = \Omega_s \dot{\boldsymbol{\varepsilon}}^s + \Omega_c \dot{\boldsymbol{\varepsilon}}^c \qquad (1b)$$

where Ω is a volume fraction. The superscripts eq, s and c refer to the homogenized (equivalent) material, the soil and the column, respectively. σ and ε are the (total) stress and strain tensors, and the dot refers to strain rate or increment. In the following it is assumed that the y-axis is in the vertical direction. The initial assumption of local equilibrium between the soil and the column material can be formulated with the following equilibrium conditions, which assure that there is no stress discontinuity between soil and column material in terms of radial and shear stresses, see also Figure 2:

$$\dot{\sigma}_x^{eq} = \dot{\sigma}_x^{s} = \dot{\sigma}_x^{c} \tag{2a}$$

$$\dot{\sigma}_z^{eq} = \dot{\sigma}_z^{s} = \dot{\sigma}_z^{c} \tag{2b}$$

$$\dot{\tau}_{xy}^{eq} = \dot{\tau}_{xy}^{s} = \dot{\tau}_{xy}^{c} \tag{2c}$$

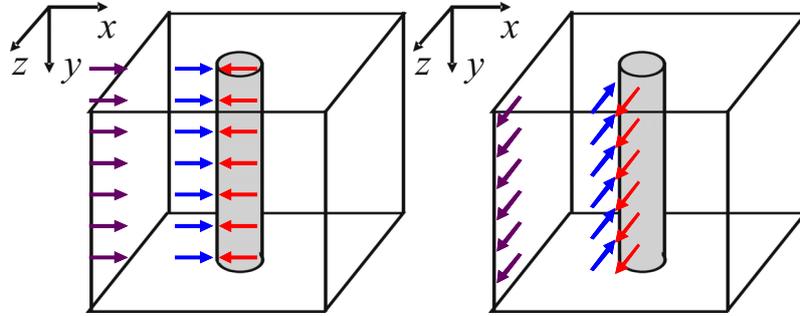$$\dot{\tau}_{yz}^{eq} = \dot{\tau}_{yz}^{s} = \dot{\tau}_{yz}^{c} \tag{2d}$$

Figure 2. Local equilibrium conditions between column and soil - radial (left) and shear stress (right) (Becker & Karstunen 2013).

Furthermore, a perfect bonding between the columns and the soft soil is assumed, which entails that no slip is permitted between the two materials. This can be achieved by adopting the following kinematic conditions, see also Figure 3:

$$\dot{\varepsilon}_y^{eq} = \dot{\varepsilon}_y^{s} = \dot{\varepsilon}_y^{c} \tag{3a}$$

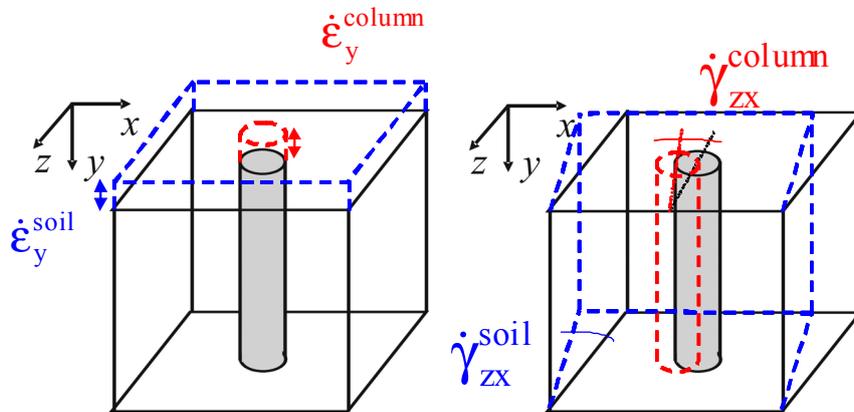$$\dot{\gamma}_{zx}^{eq} = \dot{\gamma}_{zx}^{s} = \dot{\gamma}_{zx}^{c} \tag{3b}$$

Figure 3. Kinematic conditions of perfect bonding between column and soil for axial (left) and shear strain (right) (Becker & Karstunen 2013).

The constitutive equations for the constituents can be simply described in incremental form as:

$$\left(\dot{\boldsymbol{\sigma}}^{s}\right)' = \mathbf{D}^{s}\,\dot{\boldsymbol{\varepsilon}}^{s} \tag{4a}$$

$$\left(\dot{\boldsymbol{\sigma}}^{c}\right)' = \mathbf{D}^{c}\,\dot{\boldsymbol{\varepsilon}}^{c} \tag{4b}$$

where $\mathbf{D}^{c,s}$ represent the appropriate material (elastic or elasto-plastic) stiffness matrices for the soil and the columns, expressed naturally in terms of effective stress.

Any representative constitutive law could be chosen for either of the two constituents. In the following, the S-CLAY1S model (Karstunen et al., 2005) is used to model the soft soil and the columns are represented with the MNhard model (Benz, 2007), briefly described in the following section. Considering the averaging rules (Eq. 1), the equilibrium and kinematic conditions (Eq. 2 & 3) and the constitutive relations (Eq. 4), after some manipulation the constitutive relation for the averaged material can be written as:

$$\left(\dot{\boldsymbol{\sigma}}^{eq}\right)' = \mathbf{D}^{eq}\dot{\boldsymbol{\varepsilon}}^{eq} \tag{5}$$

where $\mathbf{D}^{eq}$ (i.e. the equivalent stress-strain matrix of the homogenised material) is simply a function of the respective elasto-plastic matrices of the constituents and the volume fraction.

Constitutive Modelling

Modelling of soft soil with S-CLAY1S

The S-CLAY1S model (Karstunen et al. 2005) is an extension to the S-CLAY1 model (Wheeler et al. 2003) which allows for modelling initial anisotropy and its evolutions, as well as destructuration of bonds, as necessary for sensitive natural soft clays. The model is capable of simulating accurately the yield points and development of volumetric strains and shear strains for reconstituted clays, as demonstrated by Karstunen & Koskinen (2008). The effect of bonding, which is essential for sensitive soils, is described by an "intrinsic yield surface", which is of the same shape and inclination as the yield surface of the natural soil (Fig. 4), but with a size p'_{mi} that is related to p'_m of the natural clay by the amount of bonding x_0 ($p'_m = p'_{mi}(1 + x)$). The initial value for the amount of bonding x_0 can be estimated based on the sensitivity of the clay.

The values for the model parameters for the soft soils considered in the unit cell simulation, corresponding to Noedinge clay in Sweden are given in Tables 1 & 2. For definitions of the soil constant and state variables, the reader should refer to Karstunen et al. (2005).
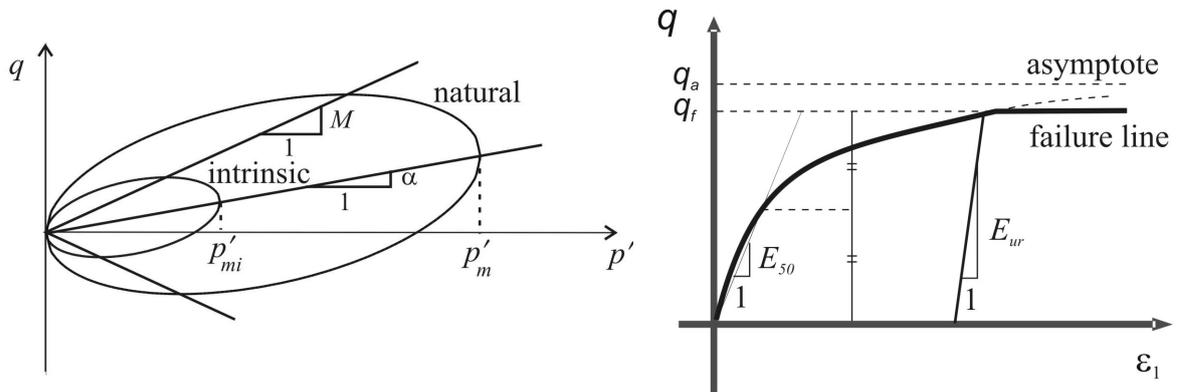

Figure 4. Yield surface of S-CLAY1S in triaxial stress space (left) and Hyperbolic stress-strain relationship in primary loading for a standard drained triaxial test (right).

Modelling of deep mixed columns

The deep mixed columns have been modelled with the MNhard model (Benz 2007), which is similar to the Hardening Soil model in the PLAXIS (Brinkgreve et al. 2010) finite element code. MNhard is formulated in the classical theory of plasticity assuming different stress-dependent stiffnesses for both elastic unloading/reloading and primary shear loading. The hyperbolic stress-strain relationship for

primary loading is defined by the secant stiffness modulus E_{50} (Fig. 4). E_{50} (and similarly E_{ur} representing unloading/reloading) are defined stress dependent:

$$E_{50} = E_{50}^{ref} \left(\frac{c' \cos \varphi' + \sigma'_3 \sin \varphi'}{c' \cos \varphi' + p'_{ref} \sin \varphi'} \right)^m \qquad (6)$$

where E_{50}^{ref} is the secant stiffness modulus at isotropic reference pressure p'_{ref}, c' and φ' are the apparent cohesion and friction angle, respectively, σ'_3 is the effective minor principle stress and the power m is defining the amount of stress dependency. Failure criterion of the MNhard model is the Matsuoka-Nakai failure criterion, which is closer to the observed failure behaviour of granular materials than the Mohr-Coulomb model, however as the predictions presented relate to serviceability limit states, they are not affected by the failure criterion.

As there are not many triaxial tests results available for characterisation of in-situ column material, the parameter values for deep mixed columns have been chosen to correspond to a series of triaxial tests performed on exhumed columns (Aalto, 2003), which were fabricated in soft Finnish clay. The input parameters for the column are given in Table 3 using the equation $E = 2G(1 + \nu')$ based on the elasticity theory where G is the shear modulus. The dry crust has been modelled as a Mohr Coulomb material, using parameter values in Table 4.

Table 1. Initial values for state parameters and K_0.

Layer	e_0	OCR	K_0	α_0	x_0
Soft clay	2.9	1.25	0.56	0.458	10

Table 2. Soil constants.

Layer	γ [kN/m³]	κ	ν'	λ	M	λ_i	β	μ	a	b
Soft clay	15	0.02	0.15	1.15	1.2	0.275	0759	30	10	0.2

Table 3. Input parameters for the columns.

Layer	γ [kN/m³]	G_{50}^{ref} [kPa]	G_{ur}^{ref} [kPa]	m	φ'	c' [kPa]	ν'	p'_{ref} [kPa]
Column	15	4444	10000	0.8	35	15	0.35	100

Table 4. Input parameters for the Dry Crust.

Layer	γ [kN/m³]	E [kPa]	φ'	c' [kPa]	ν'
Dry crust	15	750	30	6	0.15

Solution strategy and implementation into finite element code

The volume averaging technique is implemented into the PLAXIS (Brinkgreve et al. 2010) finite element code using an implicit backward Euler integration scheme. Because both constituents exhibit highly non-linear behaviour and an implicit integration scheme is used, a sub-iteration scheme was necessary (more details in Vogler & Karstunen, 2007).

Unit Cell simulation

2D and 3D unit cell FE Model and analysis
The performance of the volume averaging technique is demonstrated with unit cell simulations of floating and end bearing deep mixed columns. The soft soil is improved with lime-cement column of

10 m length and 0.6 m diameter and a centre to centre spacing of 1.0 m and 1.5 m, resulting in an improvement ratio, i.e. volume fraction, of $\Omega_c = 28.3\%$ and 12.6% respectively.

Furthermore, the performance of the volume averaging technique is demonstrated by varying the stiffness of the deep mixed column with values of $G_{ur,ref} = 10, 20$ and 50 MPa with a constant ratio $G_{50,ref} = G_{ur,ref} / 2.25$. The embankment was simply modelled as a distributed load of 10 kPa and 50 kPa above a layer of dry crust with 1 m height. Full 3D and 2D (discrete) axisymmetric finite element calculations have been compared with enhanced axisymmetric calculations using volume averaging technique. The 3D mesh contains about 4,500 15-noded wedge-elements, whereas for the axisymmetric analysis 480 15-noded triangles were sufficient, see also Figure 5. For both calculations first the initial stresses were created using a K_0-procedure. Then the columns were 'installed' wished-in place by swapping the materials, and finally embankment load was applied in undrained conditions, before simulating consolidation to a maximum remaining excess pore water pressure of 1 kPa.
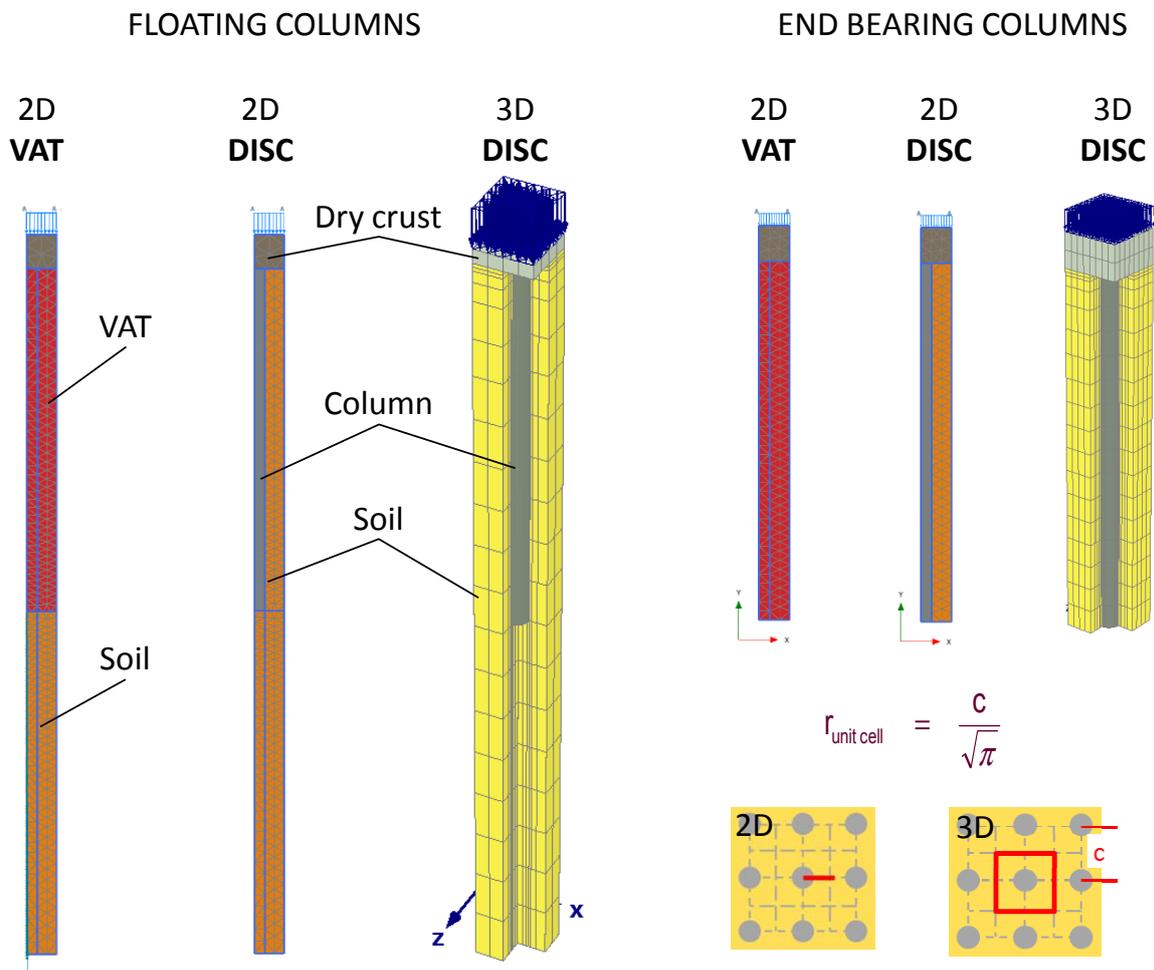

Figure 5. Unit cell FE models (2D and 3D) of floating (left) and end bearing (right) columns.

Check for local equilibrium conditions

Local equilibrium conditions (Eq. 2) between column and soft soil are fulfilled if radial stresses of column and soil are equal at a virtual column-soil interface. The radial stresses in the column (C) and the soil (S) from axisymmetric discrete modelling of the unit cell (2D-Disc) and full 3D simulations (3D-Disc) are compared with the volume averaging technique (VAT) in Figure 6. The results are presented for a volume ratio $\Omega_c = 0.283$, $G_{ur,ref} = 50$ MPa and considering loading conditions with $\Delta\sigma =$

10 kPa and 50 kPa. Local equilibrium is perfectly achieved for $\Delta\sigma$ = 10 kPa. However, the high load $\Delta\sigma$ = 50 kPa radial stresses within the columns are slightly overpredicted, whereas the stresses in the soil and VAT are almost equal. There is no major difference in behaviour between the simulation of floating and end bearing columns.
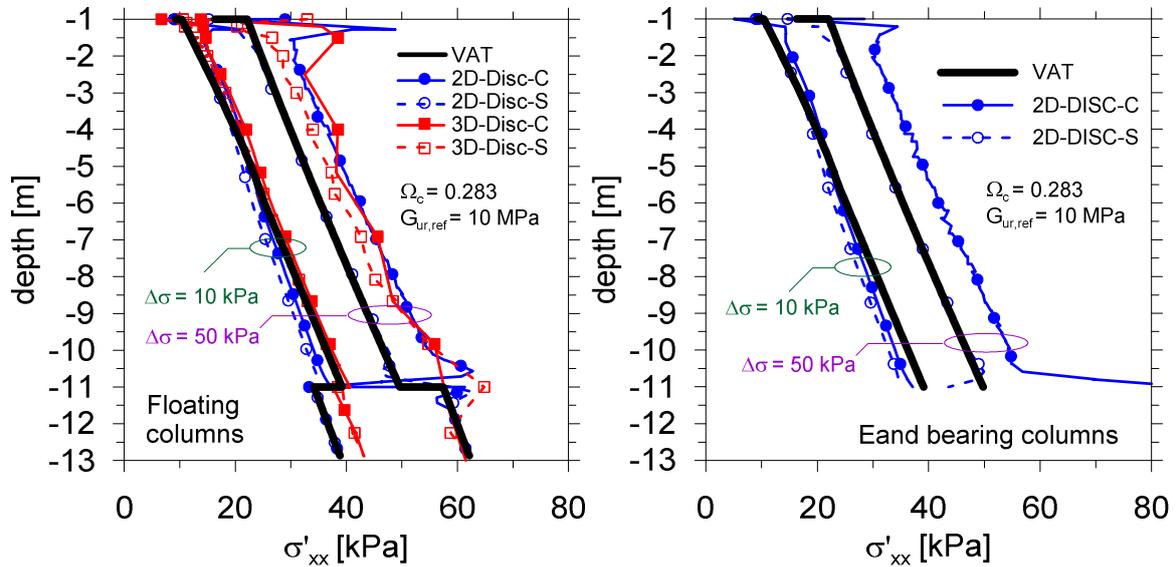
Figure 6. Radial stresses in column, soil and VAT for floating (left, after Becker & Karstunen 2013) and end bearing columns (right).

For shear stresses local equilibrium is achieved for all loading conditions, variations of input parameters and volume ratios. This is demonstrated in Figure 7, which shows the results for the higher load $\Delta\sigma$ = 50 kPa for floating and end bearing columns, as the most critical example. Scatter at the top and toe of column is due to the complex load transfer mechanisms, but its effect from practical point of view is negligible.
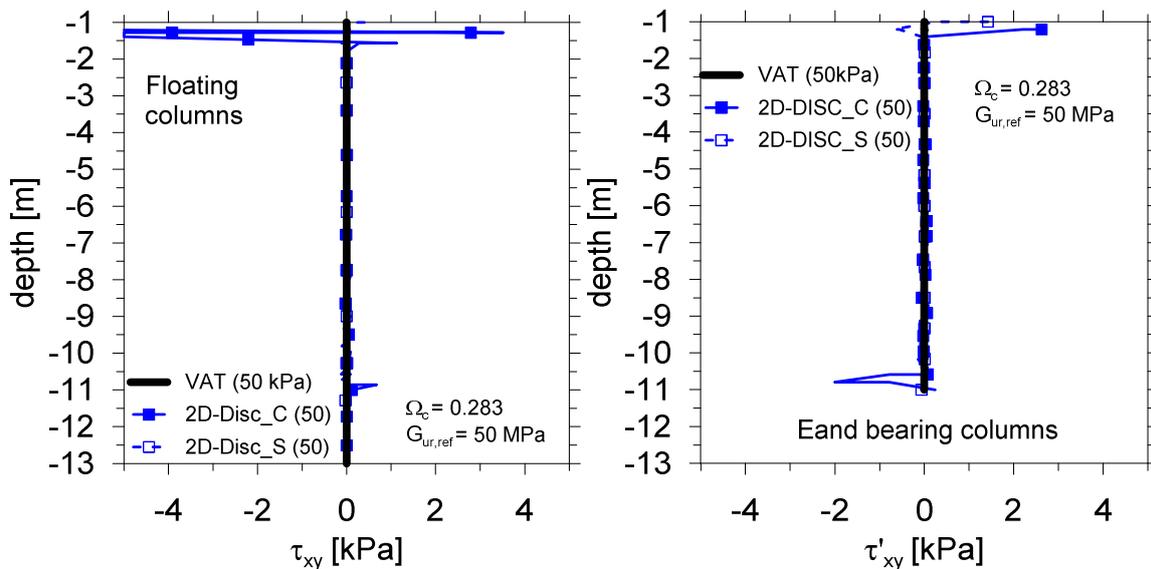
Figure 7. Shear stresses in column, soil and VAT for floating (left, after Becker & Karstunen 2013) and end bearing columns (right).

Kinematic conditions

The kinematic conditions of the volume averaging technique are based on perfect bonding between column and soft soil, i.e. no differential displacements between column and soft soil and furthermore, the radial strains have to be equal across the interface. According to Figure 8 the radial strains of the virtual column within VAT are in accordance with assumptions, even though the differences are increasing with higher load or reduced volume ratio.

Perfect bonding is achieved for the presented variations. Figure 9 shows the axial strains with a perfect match for 10 kPa load. There is a small overestimation of axial strains of VAT with increasing vertical load, but the results are still acceptable. The vertical strains in column and soil indicate perfect bonding.
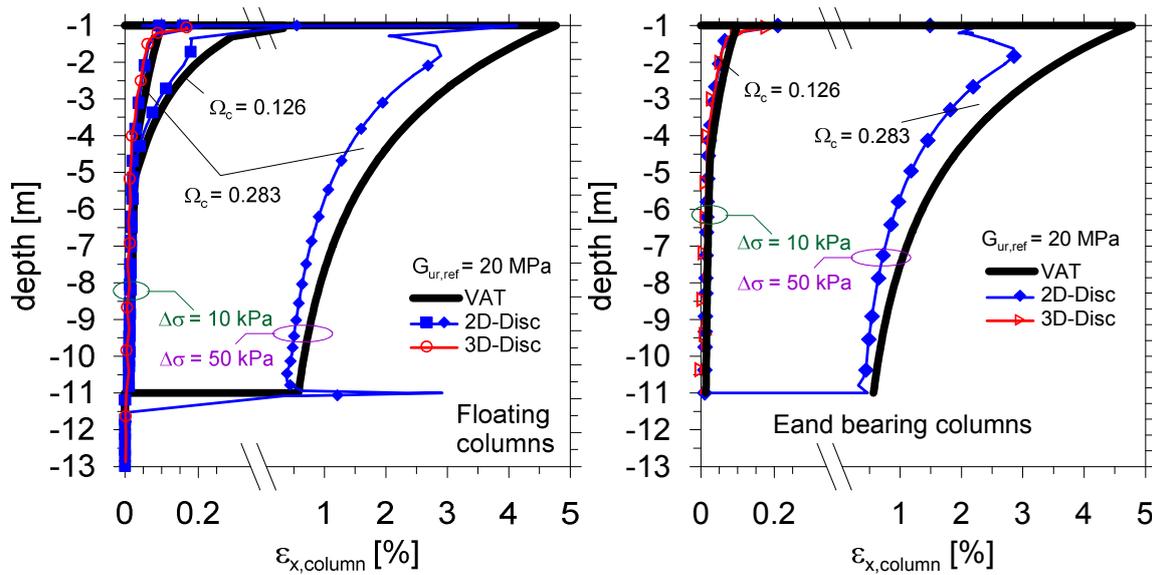
Figure 8. Radial strains in column and VAT for floating (left, after Becker & Karstunen 2013) and end bearing columns (right).
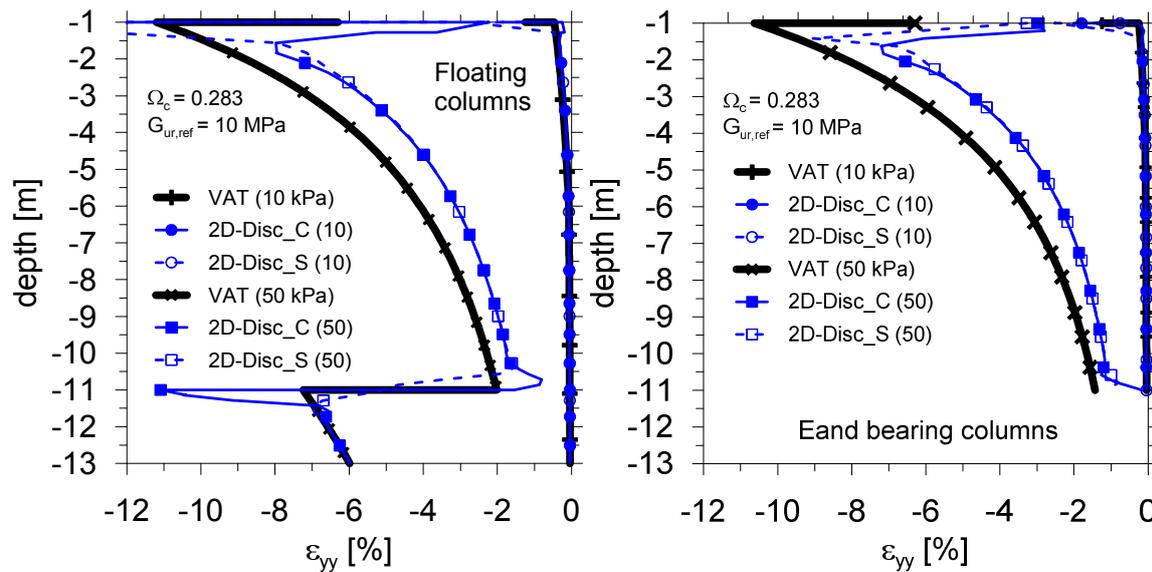
Figure 9. Axial strains in column, soil and VAT for floating (left, after Becker & Karstunen 2013) and end bearing columns (right).

Settlements

The time-dependent settlements in the centre of the column and the soil at the unit cell boundary (dashed lines) are shown in Figure 10 and in Figure 11 for 10 kPa and 50 kPa loading, respectively, with variations of column stiffness. The effect of increasing load and its influence on the equilibrium and kinematic conditions are evident with regards of the settlements as well. The VAT provides very good results for the 10 kPa load, and slightly overestimates the settlements for 50 kPa load.
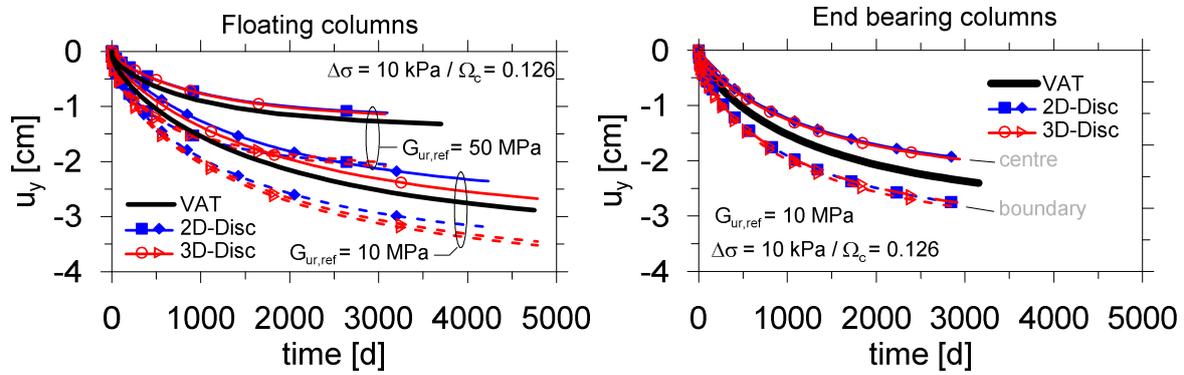

Figure 10. Settlements (load $\Delta\sigma$ = 10 kPa) for floating (left, after Becker & Karstunen 2013) and end bearing columns (right).
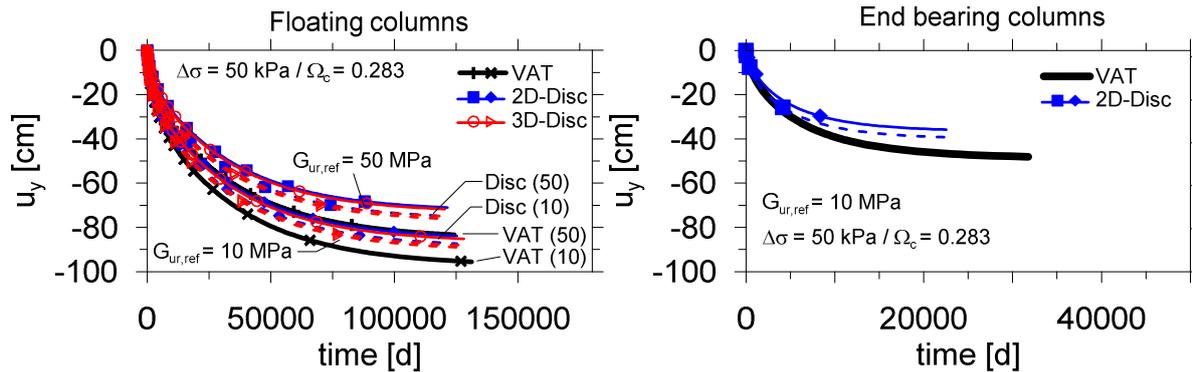

Figure 11. Settlements (load $\Delta\sigma$ = 50 kPa) for floating (left, after Becker & Karstunen 2013) and end bearing columns (right).

The time-dependent settlements are obviously affected by the different type of load transfer associated with floating and end bearing columns. As a matter of fact the vertical strains below the floating columns are already shown in Figure 9. This effect is not so relevant for low loading situation which is shown in Figure 10. But with increasing loads the settlements of the soil below the floating columns are increasing respectively.

Conclusions

The accuracy of the volume averaging technique has been studied with simulations of floating and end bearing columns in a unit cell. The technique has been verified against conventional, fully 3D analyses, where soft soil and deep mixed columns were modelled separately, as well as 2D axisymmetric unit cell. Overall, the volume averaging technique provides a good agreement with the discrete analyses. The essential equilibrium and kinematic conditions are satisfied, when applying the volume averaging technique within the typical range of applicability of ground improvements with deep mixed columns in soft soil.

Acknowledgements

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